THE LASER DUST DETECTOR

- a strategy for in-situ dust detection in planetary ice sheets

Claus Tilsted Mogensen


Ørsted Laboratory
Niels Bohr Institute for Astronomy, Physics and Geophysics
University of Copenhagen
Preface

The experimental part of this work was done at the Jet Propulsion Laboratory (JPL) in Pasadena, California, USA. JPL is a national laboratory, managed for the National Aeronautics and Space Administration (NASA) by the California Institute of Technology (Caltech). JPL is NASA’s lead center for robotic exploration of the Solar System. The laboratory has built spacecraft which have visited all the planets in the Solar System except Pluto. JPL also designs instruments that are used aboard satellites circling Earth to study the atmosphere, the oceans and land. To support this exploration, a focus of JPL is to make advances in technology and science.

During the work of acquiring my Masters in physics from the University of Copenhagen, I spent several months at JPL, first during the landed operations of the Mars Pathfinder Mission (1997) and next during the preparations for the landing of the Mars Polar Lander (MPL) (1999). The signal from MPL was unfortunately lost after entry into the Martian atmosphere. During these stays I was fortunate enough to come in contact with a group of engineers and scientists designing a probe with the objective to penetrate several kilometers of ice sheet ice while conducting in-situ analysis of the ice during descent. This probe was named the Cryobot.

I decided to apply for a Ph.D. working with this group in designing an instrument that would be able to do an in-situ study of ice sheet ice while descending through the ice. I joined the design team at JPL in August 2001 and chose to focus on in-situ detection of dust embedded in the ice as this can be very rich in paleoclimatic information. In October 2001 the first laboratory prototype of the Cryobot had been designed and was deployed at Longyearbyen Glacier at the Norwegian island of Svalbard. At this point in time I had a point design of the instrument but neither the dust detection instrument nor the Cryobot were ready for integration of the instrument into the Cryobot.

The desire was to deploy the Cryobot on Greenland, preferably near the Greenland Ice Core Project (GRIP) or North GRIP site, and to demonstrate the design of the Cryobot as well as the dust detection instrument. Deployment of the Cryobot, including the dust detection instrument, into the ice sheet would have taken place by either a melt descend performed by the Cryobot itself, a descent into one of the existing open boreholes no
longer in use, or by a descent into a hot water drilled hole. This would have been the primary data on which my Ph.D. thesis would rely.

Unfortunately, project money ran out in early 2002 and from being a group of 20 people the number quickly decreased to 3, my supervisor at JPL, Dr. Frank Carsey, Dr. Arthur Lonne Lane and myself. Thus, from having the desire to in-situ analyze the interior of glacial ice from a descending Cryobot, I had no choice but to change my focus to examining archived ice cores but I still wanted to work with the idea of having the dust detector integrated into a future probe. This change gave me a number of additional challenges.

The dust detection instrument developed in the study would be an excellent way to analyze dust embedded in ice sheet ice using an in-situ probe, but the instrument geometry is not ideal for analyzing dust in archived ice cores and thus will in no way be a competitive analytical method for detection and characterization of embedded dust in archived ice cores. One of the differences between analyzing archived ice cores and the in-situ analysis of an ice sheet is that archived ice cores have reemerging microscopic air bubbles, due to the missing overburden pressure, and in optical interrogation these air bubbles are indistinguishable from embedded dust (see e.g. Ram et al. [2000]) and therefore decrease the signal to noise ratio in the acquired data. Another issue is that the archived ice cores have a finite size. Compared to an essentially boundless ice sheet an ice core has sharp boundaries and faces that tend to diffusely reflect the laser light used to interrogate the ice. This induces unwanted reflections and stray light that in most cases degrade the signal severely. The consequences of this degradation are that measurements of the lower limit of dust concentration cannot be demonstrated and that the upper limit of the effective field of view cannot be utilized. However, the basic utility and characteristics of the approach can, and was, demonstrated.

The instrument designed to in-situ detect dust embedded in ice sheet ice, named the Laser Dust Detector (LDD) has been optimized for integration into the Cryobot. However, optimizing the performance of the LDD has been performed on archived ice cores. Nevertheless, the result is found to be satisfactory, even though deployment of the instrument into an ice sheet would have optimally tested the design.

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Claus Tilsted Mogensen
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To be able to do most of my Ph.D. at NASA’s lead center for exploration of the Solar System has, of course, been a unique opportunity. I am grateful that I got the chance to pursue a childhood dream. A dream I never would have been able to fulfill without the love and endless support from my wife Tina. I dedicate this work to you.

The experimental part of this work was done at the Jet Propulsion Laboratory (JPL), Pasadena, CA, the National Ice Core Laboratory, Denver, CO, and at the Department of Geophysics at the University of Copenhagen, Denmark. I would like to thank a series of people who have had an invaluable importance for my work.

First and foremost I would like to thank my friend and supervisor at JPL Frank Carsey for his enthusiastic and devoted support at JPL and on our stays in Denver and Copenhagen. I would like to thank him for taking the time and effort to go to snow covered Copenhagen and advising me in the data acquisition in January 2003. He has made my stay at JPL a continuous learning experience and it has been a pleasure and a privilege working with him.

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The Geophysical Department at the University of Copenhagen, has a vast experience in acquiring and analyzing ice cores from the polar ice sheets on Earth, mainly Greenland. It has been in the forefront of designing methods of ice core analysis for decades. Therefore, it was the perfect place to test the performance of the instrument designed in this work. I want to thank a number of people at the Department of Geophysics for giving support,
advice, and technical assistance, throughout my work. Especially: Jørgen Peder Steffensen and Dorthe Dahl Jensen for giving me financial support as well as technical assistance and advice about working with archived ice cores. Thank you for helping me make my stay in the US so productive and for letting me go to the North GRIP field station in the spring of 2002. Thanks to Anders Svensson and Lars Berg for assisting me in the cold storage facility.

I would like to thank the staff at the National Ice Core Laboratory, Denver, CO for assisting me in the data acquisition and the instrument testing, especially the curator Geoffrey Hargraves.

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# Contents

Preface ................................................. i
Acknowledgements ................................... iii

1 Ice Sheet Ice ....................................... 1
   1.1 Dust in Ice Sheets .............................. 5

2 The Cryobot - an Exploration Vehicle .......... 9
   2.1 Historical Overview ........................... 11
   2.2 Basic Cryobot Operation ....................... 12
   2.2.1 The Greenland Cryobot ..................... 13
   2.3 Applications for Planetary Exploration ....... 15
   2.3.1 Earth ........................................ 15
   2.3.2 Mars .......................................... 15
   2.3.3 Europa ........................................ 16

3 The Laser Dust Detector ......................... 19
   3.1 Laser Light Scattering ......................... 19
   3.2 Instrument Geometry ........................... 22
   3.2.1 The Field Deployed Instrument ............... 22
   3.2.2 The Laboratory Instrument .................. 23
   3.2.3 Constraints on Instrument Geometry ......... 25

4 Scattering of Light by Small Particles ........ 33
   4.1 Introductory Electromagnetic Theory .......... 33
   4.1.1 Poynting vector .............................. 34
   4.1.2 Polarization .................................. 36
   4.1.3 Reflection and Transmission at Oblique Incidence .... 37
   4.2 Mie Scattering .................................. 40
   4.2.1 Summary of Mie Scattering ................. 48
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2.2</td>
<td>Approximations to the Mie Theory</td>
<td>48</td>
</tr>
<tr>
<td>4.3</td>
<td>Scattering by a Size Distribution</td>
<td>49</td>
</tr>
<tr>
<td>4.4</td>
<td>Non-spherical Particles</td>
<td>52</td>
</tr>
<tr>
<td>5</td>
<td>Laboratory Experiments on Mie Scattering</td>
<td>55</td>
</tr>
<tr>
<td>5.1</td>
<td>The Single Scattering Domain</td>
<td>55</td>
</tr>
<tr>
<td>5.1.1</td>
<td>Experimental Setup</td>
<td>56</td>
</tr>
<tr>
<td>5.2</td>
<td>Experimental Verification of the Mie Model</td>
<td>62</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Experimental Setup</td>
<td>62</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Suspension Times for the Microspheres</td>
<td>65</td>
</tr>
<tr>
<td>5.3</td>
<td>Radiometry for the Laser Dust Detector</td>
<td>66</td>
</tr>
<tr>
<td>5.4</td>
<td>Calibration of the CCD Response</td>
<td>73</td>
</tr>
<tr>
<td>6</td>
<td>Dust Detection in Archived Ice Cores</td>
<td>79</td>
</tr>
<tr>
<td>6.1</td>
<td>Data Acquisition</td>
<td>79</td>
</tr>
<tr>
<td>6.2</td>
<td>Correction of the Viewing Geometry</td>
<td>83</td>
</tr>
<tr>
<td>6.2.1</td>
<td>The Scaling Function $S(c)$</td>
<td>84</td>
</tr>
<tr>
<td>6.2.2</td>
<td>Transformation of Images</td>
<td>88</td>
</tr>
<tr>
<td>6.3</td>
<td>Correlation of Images</td>
<td>90</td>
</tr>
<tr>
<td>6.4</td>
<td>Image Analysis of Archived Ice Cores</td>
<td>93</td>
</tr>
<tr>
<td>7</td>
<td>Conclusion</td>
<td>101</td>
</tr>
<tr>
<td>A</td>
<td>Microsphere Specifications</td>
<td>107</td>
</tr>
<tr>
<td>A.1</td>
<td>The Number Density of Particles</td>
<td>107</td>
</tr>
<tr>
<td>B</td>
<td>Particles found in GRIP Ice Cores</td>
<td>109</td>
</tr>
<tr>
<td>B.1</td>
<td>Number Density of Particles</td>
<td>109</td>
</tr>
<tr>
<td>C</td>
<td>Calculation of $\phi_{eff}$, $\psi_1$, and $\psi_2$</td>
<td>111</td>
</tr>
</tbody>
</table>
Chapter 1

Ice Sheet Ice

Large, rapid, and widespread climate changes were common on Earth for most of the time for which we have good records but have been absent during the few critical millennia when humans developed agriculture and industry. This record of very abrupt climate changes is clear and very convincing in the ice cores drilled from the ice sheets on Earth, mainly Antarctica and Greenland. The study of these climate changes is one of the most important scientific tasks of our time.

GRIP, the Greenland Ice Core Project, was a Danish-led, European consortium; it drilled through the Greenland ice cap at the summit between 1989 and 1992. A new drilling started in 1999 at North GRIP (NGRIP), down slope and about 300 km NW of GRIP. The drill reached bedrock in 2003 and became the longest ice core extracted from Greenland (3085 m). GISP 2, the Greenland Ice Sheet Project 2, was primarily a U.S. effort and drilled though the ice about 30 km down slope and west of GRIP between 1989 and 1993. See Figure 1.1 for locations of the major deep drilling projects on Greenland.

Ice cores drilled and extracted from the ice sheets, and analyzed in laboratories around the world provide information of inestimable value in the reconstruction of past climates on the Earth (see e.g. Alley [2000] for a review). These ice cores are highly detailed and often contain uninterrupted climate records extending back hundreds of thousands of years. In general the Antarctic ice sheet contains information covering the past 420,000 years whereas the ice sheet on Greenland, where the yearly accumulation is much higher, contains information up to the past 110,000 years. The information contained in the extracted ice cores include:

- Temperature: The composition of the stable oxygen isotopes ($^{16}$O and $^{18}$O)$^{1}$ in snow falling on an ice sheet is mainly a function of the temperature at the time of deposition. The oxygen isotope ratio$^{2}$ or $\delta^{18}$O value, measured using a mass spectrometer

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$^{1}$All other oxygen isotopes have either very short half lives or appear in nature in only trace amounts.

$^{2}$The definition of $\delta^{18}$O is $[(^{18}$O/$^{16}$O)$_{\text{ice}}-(^{18}$O/$^{16}$O)$_{\text{SMOW}}]/(^{18}$O/$^{16}$O)$_{\text{SMOW}}$ expressed in per mille.
on melted core samples, provides a proxy of the temperature at the time the ice was deposited as snow. Isotopically, the water vapor in an air mass starts out a little lighter than in the ocean, because isotopically lighter water ($\text{H}_2^{16}\text{O}$) has an easier time evaporating than $\text{H}_2^{18}\text{O}$. When the water vapor has cooled down and formed clouds, the first raindrop to fall from this cloud is, on average, isotopically heavier than the water vapor left in the air mass, as heavier water condenses more easily. As rain and snow are removed from an air mass, the precipitation is always isotopically heavier than the vapor it leaves behind. As the air mass becomes colder and colder, the remaining vapor becomes lighter and lighter, and the precipitation also becomes lighter and lighter as the air mass runs out of heavy isotopes lost to precipitation. This is the process behind paleothermometry. If the isotopic composition of a sample of water can be measured, the temperature of the water, at the time of precipitation, can be inferred. One method to calibrate the 'isotopic thermometer' is to measure borehole temperatures and compare the measurements with predicted temperatures from isotopic analyses. To calibrate the isotopic thermometer, basically only three parameters need to be varied, namely the change in

where SMOV is Standard Mean Ocean Water.
isotopic ratio needed for a one-degree change in temperature, the temperature at which precipitation would match the composition of ocean water (SMOW), and how much geothermal heat is supplied to the ice sheet.

- Wind Patterns: The characteristics of microparticles in ice cores (shape, size distribution, composition, and morphology) are representative of the microparticles present in the lower atmosphere during the time of deposition. Therefore by careful analysis of the microparticles found, the point of origin of the particles can be inferred. A change in particle characteristics vs. depth in the ice core thus shows a change in the atmospheric content of microparticles; the change may be due to a change in the origin of the microparticles. Reconstruction of atmospheric circulation patterns and their changes over time from chemical indicators and dust sources provides new insight into the large and rapid changes documented in ice cores.

- Gas composition: Snow falling on the surface in areas with no melting is deposited on the surface and covered by subsequent snowfalls. As more and more snow is accumulated on top, the underlying snow is compressed, first into firn and eventually the density increases and the firn becomes ice. As the snow is compressed, the air filled spaces between the ice crystals become sealed off and air bubbles are formed in the ice. Ice cores therefore can be analyzed for the properties of the air trapped in the ice as long as no chemical changes have occurred. These bubbles are samples of the atmosphere and contains information about the chemical composition of the atmosphere in the past.

- Seasonal variations: Dust content, chemical composition, and isotopic ratios all change with the seasons. In central Greenland, snow falls frequently throughout the year but is dustier in spring. The sun never sets in the summer and never rises in the winter. Winter snow is thus buried without experiencing sunshine, but summer snow is heated by the summer sun and the top centimeter or two can be heated as much as 3°C warmer than the air above. Some of the snow in the warmer layer sublimes and when it gets colder in the evening the air above the ground condenses and forms frost on the surface. The daily heat and nightly cold of the summertime thus turns one cm of wind-packed, fine-grained snow into two cm of coarse-grained, low-density snow called hoarfrost. As the transformation of regular snow to hoarfrost is driven by the summer sun, the summer and winter snow are different. The distinction between summer and winter snow is easy in the top kilometer or so by the difference between summer snow with large air bubbles and more densely packed winter snow. It is more difficult to discern the seasonal snow when the air bubbles disappear into clathrates below 1200-1400 m (at Greenland).
Here the pressure becomes so great that the air in the bubbles begins to dissolve into the ice. The hexagonal arrangement of water molecules in the ice right next to an air bubble change to a roomier cubic structure, and the air molecules are positioned into the spaces in the centers of these structures. The air bubbles disappear, replaced by bubble-sized pieces of an ice-air mixture, called a clathrate. The clathrate, which looks and acts almost like ice, forms because the air takes up less space in the clathrate than in the air bubble. As clathrates replace bubbles, the ice becomes completely transparent, see Ikeda-Fukazawa and Hondoh [2003] for a more detailed description. In newly extracted ice cores the bubbles begin to reappear after a few month as the clathrates break down due to the missing pressure.

Another example of annual differences in an ice core is the amount of hydrogen peroxide. $\text{H}_2\text{O}_2$ is created in the atmosphere by a chemical reaction that requires ultraviolet light. There is less ultraviolet light in the winter than in the summer at high latitude sites such as Greenland. Thus, measurements of hydrogen peroxide dissolved in the ice also provide a good annual cycle indicator. We already talked about the change in temperature vs. depth which obviously also can be used to discern annual layers as the winter surface temperature is lower than the summer surface temperature. Another widely used method is to measure the electrical conductivity of the ice (ECM) [Taylor et al., 1997; Wolff et al., 1997]. These measurements are closely linked to the acidity of the ice. Most snow and ice are naturally weak acids. Chemical reactions in the atmosphere involving dimethyl sulphide (a chemical produced in greater quantities during the summer months by marine algae and phytoplankton) result in production and deposition of sulphuric acid, and carbon dioxide in the atmosphere reacts with water and snow and are deposited on the ice sheet as carbonic acid. Acidic ice is more conductive than neutral ice and the ECM shows an annual cycle where the conductivity of the ice is higher in the summer than in the winter.

- Volcanic horizons: Sulphur is often blasted into the atmosphere by volcanic eruptions. Therefore, ECM measurements of the ice cores sometimes shows a peak in the conductivity at the depth corresponding to the time shortly after a volcanic eruption. A more reliable method of detecting volcanic eruptions from the ice cores however, is to measure sulphate directly. Early ice core studies showed that eruption of the Icelandic volcano Laki in 1783 dumped more sulfuric acid on Greenland than any other event that occurred within several centuries of Laki, producing a spike in ECM and chemical records that stands far above surrounding peaks. The Laki peak is a time marker that allows all cores in Greenland to be correlated, and allows dating to be checked. Other volcanic eruptions have been used as time mark-
1.1. DUST IN ICE SHEETS

ers, including the eruption of Mount St. Helens in 1479 and the year 79 eruption of Vesuvius. Historical records of known volcanic eruption thus help date the ice. Dust layer counting, ECM, and isotope ratio cycling completes the dating as they can be accessed much further back in time. For example Ram et al. [1996] found volcanic ash in the GISP2 ice core at 2464 m depth and correlated this ash with the Z2 ash layer found in Atlantic sediment cores, with ash in the Dye 3 ice core from Greenland, and with ash found in the GRIP core from Central Greenland. The mineralogy is very similar to that of ash from the Icelandic Tindfjallajökull volcano and suggest that this is the source volcano. This way dating of multiple ice cores can be equalized by the same event and due to the accurate dating of the GISP2 ice core (by counting dust layers) Ram et al. [1996] determined that the eruption that produced the ash occurred $57,300 \pm 1,700$ years before present.

The vast amount of information deduced from analysis of the Greenland ice cores tell a story of climate change with very abrupt changes in temperature when the Earth experienced large, rapid, regional-to-global scale climate oscillations through most of the last 110,000 years. These events represent large climate deviations that probably include change in temperature of many degrees Celsius, twofold changes in snow accumulation, large changes in how much wind-blown dust and sea salt were carried by the atmosphere, and large changes in methane concentration. These rapid and abrupt changes commonly occurred over decades or less. The ability to count annual layers in the cores well into the glacial period and probably through 110,000 years is crucial in answering questions about the timing of the glacial periods. The use of volcanic markers (such as dust and gases), atmospheric-oxygen isotope ratios, and ocean sediment records to determine the ages of ice cores greatly extends the ability to map climate changes.

1.1 Dust in Ice Sheets

Changes in the properties of the insoluble microparticles, or dust, found in ice cores as a function of ice core depth allow for an interpretation of temporal climate change and are an important tool in paleoenvironmental reconstruction.

The dust concentration in ice from the glacial period is much higher than in the Holocene ice. Typically up to 100 times higher for the Last Glacial Maximum (LGM) [Steffensen, 1997]. The huge variation in dust concentration is believed to be caused by a change in the atmospheric transportation mechanism and possibly a change in the source of the dust. An increase in the frequency of storms or at least in their storminess over the continents, and a more effective atmospheric circulation in the glacial period is believed to be a large factor in the increased dust deposition. Also an increase in the source area,
namely the exposed seabeds arising from a lowering of the ocean level, is thought to be a factor as well as an increased dryness of desert regions. Dust deposition is mainly due to the fact that atmospheric particles act as condensation nuclei for snow deposited on the ice sheets, as well as settling directly out of the atmosphere.

Alley [2000] gives an overview of dust found in extracted ice cores from Greenland. The bulk of the dust is windblown and the origin is continental. The annual variation of dust concentration peaks in early spring and has a minimum in late fall/winter. This is true for the glacial period as well as for the Holocene.

Insoluble continental dust is composed mainly of aluminosilicates and a small fraction of calcium carbonate. There is a good correlation between the concentration of dust (Al), Ca, and non sea salt Mg in all climate periods. The mass ratio between dust and Ca is practically constant throughout the different climatic periods. Steffensen [1995, 1997] concludes that the composition of the source has not changed and the high concentrations of Ca$^{2+}$ and Mg$^{2+}$ in the glacial periods are explained by a generally higher supply of dust.

A range in dust particle sizes is found in the ice from ice sheets. The smaller particles with radius $a$ in the range $0.4 \mu m < a < 2.5 \mu m$ can be described by a log-normal size distribution. The larger particles $(2.5 \mu m < a < 7.0 \mu m)$ however, can not be described by a simple change in the parameters of the log-normal size distribution. A different distribution is therefore used to describe the larger particles, namely the Junge distribution. A typical mass fraction of smaller particles to larger particles is 5-8% Steffensen [1997]. The size distribution of the smaller particles in the Greenland ice resembles that of the continental dust in the upper troposphere at $40^\circ$ N latitude. The smaller particles found in the ice are therefore assumed to be windblown continental dust. Particles coming from a single source and fed to the atmosphere by a single mechanism usually have a narrow log-normal size distribution, whereas particles with a broader distribution indicate a combination of mechanisms. A main conclusion of Steffensen [1995] is that the size distribution of the bulk of dust found in ice cores from the Dye 3 drilling in Greenland doesn’t change between different climatic periods.

Significant difference, found by Steffensen [1995], exists between the smaller and larger particles. In addition to the change in the number concentration of the two fractions, the annual change in concentration of dust in the ice from the Holocene is only a change in the concentration of the smaller particles. The concentration of the larger particles remains nearly constant. The much higher concentration of particles in the LGM is a result of a much higher concentration of smaller particles. The concentration of smaller particles in the LGM was 40 times than that of the Holocene, whereas the increase in the concentration of larger particles only was tenfold.
1.1. DUST IN ICE SHEETS

The small particles are believed to have been transported from the source to the Greenland ice sheet in the lower troposphere as the size distribution shows a rather narrow log-normal distribution. This narrow distribution probably means that no mixing has occurred and that the methods of transportation have left the particle distribution largely unmodified. The annual variation of the concentration of smaller particles also reflects that the particles have been transported in a part of the atmosphere that changes with the season, as does the lower troposphere.

The fraction of larger particles is believed to have been transported from the source to the Greenland ice sheet by the upper troposphere as this fraction does not contain an annual variation. It is therefore believed that the fraction of larger particles stems from the background aerosol in the upper troposphere that doesn’t have a seasonal variation in wind pattern. The natural background aerosol is the result of a statistically equilibrium as the aerosol is constantly fed from sources and constantly depleted by sinks.
Chapter 2

The Cryobot - an Exploration Vehicle

On Earth, the most common approach to the study of ice sheets is to return a mechanically drilled ice core to the laboratory where the composition and structure can be examined. This way a full suite of interrogation techniques can be applied to the ice cores extracted from the ice sheet. Moreover one can go back to the same core for reexamination after initial analysis has been done. Some of the drawbacks of this technique are that it’s a very expensive and time consuming method. Another drawback is that the ice core storage techniques used are unable to prevent temporal degradation of the cores. Specifically, examination of extracted ice cores shows microscopic air bubbles reappearing in the cores after a short time, see e.g. Ram et al. [2000]. These bubbles scatter light in much the same way as dust particles do. In non-destructive optical analysis of the cores it is very difficult to distinguish the scattered signal from small embedded dust particles and from microbubbles. This makes non-destructive optical dust analysis on extracted ice cores a complicated task.

The less-common thermal or Philbert probe is an excellent way of acquiring in-situ data at a low cost and fast rate (compared to the full core extraction). The Philbert probe is a non-recoverable vehicle designed to carry instrumentation to any depth in polar ice sheets by melt penetration. The probe is controlled and powered from the surface through an internally stored tether which pays out of the advancing probe and becomes anchored in the refrozen water above. See Kelty [1995] for a review. Such a probe was successfully used to penetrate over 1 km of ice in Greenland [Philberth, 1976]. Aamot [1968] continued the development, emphasizing on the vertical stability of the probe and hence its ability to descent in a straight vertical line.

The following chapter contains a description of a modification of the Philbert probe, namely the NASA-JPL Active Thermal Probe. From 1999 to 2002, JPL developed and
demonstrated a modified thermal probe design named the Cryobot. The Cryobot is a cylindrical, tethered, robotic platform that will descend using passive heaters until a thin meltwater envelope is maintained around the probe, thus allowing the water reservoir in the Pump Bay to fill, see Figure 2.1. This is expected to occur some meters below the surface\(^1\). Filling of the Pump Pay will trigger a water jet, which enhance heat transfer and move dust and other particulates away from the nose of the probe. To approach the robustness of the hot water drill [Engelhardt et al., 2000], JPL added an internal water jet to the Aamot-style probe. The resulting Cryobot (see Figure 2.1) is vertically stable and tolerant of embedded particulate layers, as verified in numerous laboratory tests [Zimmerman, 2001] and an actual field deployment at the Longyearbyen glacier, Svalbard, Norway.

The Cryobot melts ice passively, through resistive heating of its nose, as well as actively, through forcing a jet of hot water against the ice, through a nozzle in its nose. The Cryobot is thus a Philberth probe augmented with a tiny hot-water drill and suspended from its tether, stored on board. The mass of the probe nose and Pump Bay hanging

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\(^1\)The production of a stable body of liquid meltwater assumes ambient pressure and temperature capable of sustaining liquid water.
2.1. HISTORICAL OVERVIEW

Field testing of the prototype Cryobot was performed in collaboration with the Norwegian Polar Institute and the Norwegian Space Agency. The probe successfully penetrated 23 m in glacier ice at a rate of 0.6-0.7 m/hr, using 750 W. Moreover, the probe successfully penetrated multiple layers of dust and debris. The deployed probe was primarily designed for laboratory testing purposes and had not been constructed specifically for field work, and its eventual failure was not unexpected. Nonetheless, the probe degraded gracefully with progressive failures in pumping and heating subsystems, and still was able to operate as long as power could be transmitted from the surface.

The primary goals behind the development of the JPL Cryobot were a desire for in-situ scientific access to the deep glacial domains of Earth, Mars, and the Jupiter moon Europa. System requirements for these three generalized sites range broadly, but in all cases the requirements include descent and scientific profiling through several kilometers of ice, communication of significant data volumes (especially due to imaging), knowledge of location, attitude and orientation of the probe, and the ability of performing non-contaminating exploration.

We note that at this time, and for the foreseeable future, ice cores will not be replaced by instrumented probes as primary data sources for paleoclimatology.

2.1 Historical Overview

To assess the applicability of data that might be acquired from the inside of a borehole, a JPL team, in collaboration with a science team at California Institute of Technology (Caltech) obtained images from cameras lowered into deep holes in Antarctica created by a hot-water drill [Engelhardt et al., 2000; Carsey et al., 2002]. In 2000 JPL designed, built and deployed this so called Ice Borehole Camera (IBC) [Behar et al., 2001]. The IBC is a relatively simple system. It consists of a tethered pressure housing and 2 sets of cameras and lights, one looking downward and one looking sideward, connected to the surface by a power cable and an optical fiber cable for data transport. It was deployed to study the effects of basal processes in Ice Stream C, Antarctica; the results were striking and different from expectations such that the data are still being examined [Behar, 2003]. As shown in Figure 2.2, variations in ice texture and well defined layers of trapped particles, presumably reflecting upstream basal processes, are clearly seen. In summary [Behar, 2003], ice lenses comparable to those found in permafrost were observed; isolated clasts of basement debris suspended in clean ice were observed; debris was found far higher in the ice sheet (25 m) than expected and a very interesting subglacial "lake" of 1.4 m depth was penetrated by the IBC. These results indicate (as had been suspected [Behar,
2.2 Basic Cryobot Operation

The suspension of the Cryobot from it’s tether, frozen into the ice, allows the probe to follow the local gravity vector and maintain a vertical descent trajectory. The onboard computer performs all power control, sensor functions, vehicle control (i.e. navigation), and science instrument operation. In the near-surface firn (ice with interconnected voids), the passive heaters will be required to initiate descent due to the porosity of the firn and subsequent loss of melt-water. In the firn the Pump Bay will most likely not be filled with melt water. However, in the deeper ice a thin melt water envelope will be maintained around the probe allowing the Pump Bay to fill. The on-board sensors will detect the water level in the Pump Bay and provide the necessary signal to activate the water jet. The operation of the water jet not only facilitates efficient melting, but also moves particulates away from the nose of the probe. The melt front generated by the passive nose heaters and the water jet is sensed by the forward looking acoustic ranging sensors. The distance to the melt front must be maintained to keep the diameter of the melt column just slightly larger than the vehicle diameter. As the distance to the melt front changes as a function of ice temperature, signals from the acoustic ranging device are fed back to the onboard computer which in turn either reduces or increases a combination of heating
2.2. BASIC CRYOBOT OPERATION

and tether pay-out rate. This closed loop control mechanism maintains an efficient melt rate by minimizing the thickness of the liquid meltwater envelope.

Vehicle descent control uses the onboard inclinometer (2-axis tilt sensor) data coupled with heating of the passive heaters. As the nose is separated into 4 independently heated segments activation of 2 neighboring heater segments will enable crude steering. Thus, activation of two of the nose passive heaters initiates growth of a convective melt layer on that side of the probe. Activation of the opposing heaters in the hull (the hull heaters) initiates growth of a convective melt layer on the opposite upper surface of the probe. This opposing nose and hull heater operation allows the probe to slowly rotate away from its vertical descent vector, or move back to the vertical after a turn.

2.2.1 The Greenland Cryobot

The Greenland Cryobot\(^2\) consists of three main chambers, the Nose and Pump Bay, the Pressure Vessel, and the Tether Bay, as shown in Figure 2.1:

- The Nose and Pump Bay include the four-quadrant resistance heaters to heat the nose, the water heaters for the jet, and the jetting pump to force the water against the ice. It also includes sensors to measure water temperature, ambient pressure, basic water characteristics and the presence of water in the bay. Additional thermometers are mounted in other locations of the Cryobot and space is reserved for short range ultrasonic sounders to be used for ranging (not included at the Svalbard field deployment). Furthermore the Pump Bay will be able to contain additional ‘wet’ science instruments that examine meltwater samples to assess chemical and/or biological signatures contained in the local environment of the ice sheet.

- The Pressure Vessel sits above the Pump Bay and consists of an Instrument Bay and an Electronics Bay. The Electronics Bay contains the system control electronics and the Instrument Bay contains ‘dry’ science instruments. The ice and the thin meltwater envelope surrounding the probe are observed through a window in the hull, using a system of cameras and lights. In the Pressure Vessel are additional hull heaters. The Laser Dust Detector, designed and tested in this work, will be integrated into the Instrument Bay.

- The Tether Bay sits behind the Pressure Vessel and contains the tether and tether actuator. It should be noted that in temperate ice, where it takes a significant period of time for the melt hole to refreeze, the tether can be deployed from the surface.

\(^2\)Even though the first field deployment happened at the island of Svalbard, Norway, the first plan was to deploy the Cryobot at the Greenland ice cap.
CHAPTER 2. THE CRYOBOT - AN EXPLORATION VEHICLE

The tether actuator controls the rate of descent of the probe during melting. The tether deployment device is the point of suspension of the Cryobot. In the tether are copper lines for power and optical fibers for data transmission. Monitoring of the tether release is the primary measure of downward displacement whereas the photographic records and the pressure data serve as backups.

There is at present only one working design of the Greenland Cryobot. Other Cryobot systems under consideration are the "Vostok Subglacial Lake Explorer", the "Mars CryoScout”, and the tentative "Europa Cryobot”, the latter 3 only on the drawing board at the current point in time (year 2003).

It should be noted that several technical and mechanical design issues have not been solved at this point in time. They include

- Miniaturization: For extraterrestrial exploration miniaturization and weight reduction are important factors. Weight reduction minimizes the launch cost and miniaturization reduces the power necessary to run the probe. The Mars CryoScout working model has been miniaturized to an 8 cm diameter, 72 cm long probe. To be able to send the probe to Europa even further miniaturization must be implemented.

- Power source: On Earth power can be supplied by a diesel generator on the surface. On Mars or Europa power must be generated via a combination of batteries and solar cells or via a miniature radioisotope thermal generator (RTG). The appropriate RTG is a major design task in itself. Solar cells, on Mars, must produce a constant high level of power with potential ice and dust deposited on them.

- Planetary protection: On Mars and Europa, as well as for the Lake Vostok exploration, planetary protection is of great concern. Methods must be developed to clean and possibly sterilize the probe to avoid contamination of the environment.

- Communication: Transfer of power from the lander station down to the Cryobot and transfer of data from the Cryobot to the lander must be efficient. This can be done on Earth, and possibly on Mars, with a combination of a copper cable for power transfer and fiber optics for data transfer. On Europa however, the power source will most probably be an RTG, integrated into the Cryobot itself. This way no power has to be transferred from the surface to the probe. Data transfer could be managed by deploying small relay stations as the probe descent the ice sheet.

It is recognized that some of these design tasks will be very difficult challenges for which the solution lies well into the future.
2.3 Applications for Planetary Exploration

By profiling an ice sheet, a Cryobot can determine physical properties such as profiles of temperature, conductivity, opacity, and the amount of dust in the ice [Carsey et al., 2003]. It can measure chemical properties such as the redox profile, pH, and dissolved ions and gases as well as isotopic ratios indicative of climate change and potentially the distribution of trace biomarkers. Such a Cryobot is the obvious means to explore the icy shell of Europa, where it may be necessary to penetrate below a surface zone to reach an environment that is not continuously damaged by radiation and impacts. On Mars, it may well be the simplest way to explore a significant vertical range of sub-surface deposits, covering millions of years, in the ice caps.

2.3.1 Earth

On Earth, Cryobots may be used to study paleoclimate and ice dynamics on Greenland and Antarctica and perhaps the putative extremophile biology of Lake Vostok. Recent results from Greenland [Thomas et al., 2000; Dahl-Jensen et al., 1997] and Antarctica [Joughin and Tulaczyk, 2001; Petit et al., 1999] have clearly indicated that the deep ice contains significant information regarding ice dynamics and paleoclimatology [Hoffman et al., 1998; Alley et al., 1997].

Lake Vostok is buried under almost 4 km of ice under the Russian Vostok field station. The thick ice sheet above the lake acts as a blanket, shielding the lake from cold temperatures on the surface allowing geothermal heat to keep the water liquid [Siegert et al., 2001]. The lake itself has never been sampled - the deepest ice sample acquired at this point in time (2003) is accreted lake-water ice from 100 m above the liquid surface. Analyses of the ice samples indicate that the lake is a suitable habitat for simple lifeforms [Siegert et al., 2001]. Clearly, it provides an excellent simulator for another extensive liquid water-body - namely the sub-ice ocean on Jupiter’s moon Europa. Thus it’s possible that Europa and Lake Vostok may share a number of remarkable properties, including a thick covering of ice (several kilometers) over liquid water, an environment where microbial life may have exploited unique habitats and subterranean waterways accessible to remote sensing via radio sounding techniques and in-situ observations by means of melting probes, like the Cryobot.

2.3.2 Mars

On Mars, a melting probe could be used to profile the ice cap to study planetary history in general and the presence of cosmogenic and other organic compounds in particular. The polar caps of Mars, like those on Earth, have long been considered as archives of
paleoclimatic information. Thus a subsurface profile of ice and dust properties in the caps is possibly the only data source on Mars’ recent climate [Murray et al., 1972].

Polar cap exploration is of obvious value in understanding climate change. The Mars North Polar Cap is topographically quite similar to the Greenland ice cap although its temperature profile, microstructure and mechanical properties are probably significantly different. Finally, the north cap rests on the seafloor of the putative early Mars ocean.

At present we have no information about the microscopic character of the ice that makes up the Mars North Polar Cap. There are thermodynamic analyses indicating that the cap is made up of water ice and modeling results indicating that the density structure has no low-density surface layer [Arthern et al., 2000]. The appearance of the surface is not a useful indicator of the nature and density of inclusions or horizons. The Greenland chronology has been established partly by annual layer counting, but on Mars this is unlikely as the accumulation rate, while unknown, is thought to vary between 1 mm/sol and .01 mm/sol with some arguments that the accumulation may go through long periods of time with a negative value of accumulation rate. There are clearly visible surface layers spaced 10-30 m apart; their presence is most likely due to climate changes on Mars. There is little argument that the analysis of dust distribution in the Martian ice, including the character of these layers, is crucial to understanding Mars’ climate.

The key to obtaining suitable data on the properties of Mars ice caps is in the design of an exploratory polar cap in-situ mission and its instrumentation, of which the optical systems for the visible stratigraphy and dust character are significant. In taking steps to obtain the optical data we begin by noting that the polar cap ice may be clear in the fashion of deep ice sheet ice from Greenland or Antarctica, or it may have sufficient dust and bubbles to be opaque, or somewhere in between. A Cryobot instrumented with a Laser Dust Detector and an additional light source would be able to profile the ice column either by penetrating a rather clear ice sheet or by imaging the wall of the melt hole of an opaque ice sheet.

2.3.3 Europa

The existence of an ocean under the ice [Stevenson, 2000] and the possibility that it may harbor life is a fundamental motivation for the exploration of Europa. The global ice cover holds clues to an old, deep, salty, ice-covered ocean indicated most convincingly by the Galileo spacecraft [Kivelson et al., 2000; Stevenson, 2000]. Some models for the origin of life on Earth suggest a submarine origin at hydrothermal vents. Within this framework there has been speculations of life in the ocean on Europa, which appear to have at least three ingredients considered essential for life: water, energy, and the necessary chemical elements.
2.3. APPLICATIONS FOR PLANETARY EXPLORATION

The scientific goals of landing on the surface and exploring the sub-surface would include determining whether the moon do indeed have a subsurface ocean; mapping the organic compounds and other chemicals of biological interest; and possibly determining the thickness of the ice cover.
Chapter 3

The Laser Dust Detector

3.1 Laser Light Scattering

Laser Light Scattering (LLS) is a powerful, fast, and non-destructive way of optically interrogating embedded microparticles in an ice core. Under the non-destructive assumption it is obviously assumed that the LLS is done on solid ice (and not on a melted sample) and that the power and frequency of the laser beam is such that any organic or biological compounds in the ice remain unaltered by the examination.

Many investigators have used LLS to acquire information about the dust embedded in the ice. The method has been successful, in part, because the particle size distributions in ice cores have been very constant in time. Even during a change by a factor of 100 in concentration the size distribution is only slightly changed [Steffensen, 1997]. Another reason the optical interrogation of dust has been successful is that the overall mineralogy of the dust particles appear to be more or less constant, i.e. the index of refraction of the particles doesn’t change much with depth or particle size, an important factor when analyzing the experimental data.

Shallow polar ice (0 m to $\sim$1200 m on Central Greenland) contains numerous visible air bubbles that are remnants of air trapped between snow crystals during the deposition of snow on the ice sheet. Air bubbles are excellent scatterers of light and can dominate the scattering when analyzing the shallow ice with a laser. Therefore meltwater analysis is a good way to quantify the concentration of dust in an optical interrogating technique of the shallow ice. The drawback of this technique is that it’s a destructive analysis and quite time consuming. At depths greater than about 1400 m (at Summit, Greenland) however, air bubbles are no longer present in the ice when the ice is first retrieved. The air bubbles have disappeared by the formation of clathrate hydrates. Therefore, interrogation of solid ice is a faster and non-destructive method and is thus the preferred LLS technique in the deeper bubble-free ice.
Hammer et al. [1978] was the first to propose the 90° laser light scattering technique. In Hammer’s method, laser light is passed through a sample of ice meltwater, and the intensity of light scattered at 90° relative to the incident beam direction is detected by a photomultiplier tube (PMT). According to Hammer, the intensity of 90° scattered light is directly proportional to the mass of particulate matter suspended in the meltwater. Hammer et al. [1985] later used the LLS technique (from meltwater) to measure the dust concentration profile of the Dye 3, Greenland, ice core. They analyzed small melted samples of the ice, and found, by comparison with Coulter counter measurements, a linear relationship between the intensity of the scattered light and the concentration (number of particles per milliliter) of dust particles larger than 0.5 µm.

Royer [1981] verified the 90° LLS technique by using instrumental neutron activation analysis (INAA) of aluminum to measure the dust concentration in polar ice. Royer used dust recovered from filtration of polar ice meltwater and analyzed it via INAA of aluminum to determine the concentration of dust in the ice as aluminum is believed to be a stable representative of the continental dust present in polar ice [Royer, 1981; Ram et al., 2000]. Royer carried out extensive measurements on ice from Dome C, Antarctica, to evaluate the technique and found a good linear relationship between the 90° scattered light intensity and the mass of insoluble particles suspended in the meltwater.

Ram and Illing [1994] developed, in an effort to assist with the dating of the GISP2 ice core, a 90° meltwater LLS apparatus. The setup was mounted on a linear translation stage and would melt a small section of the ice, transfer the meltwater to an optical cell for visual inspection before the melt water was finally transferred to a light tight optical cell for LLS analysis. The importance of the two cell system was to make sure that the final optical cell was completely filled as an air filled cell could produce an unwanted scattering peak. The procedure was only semi-automatic as a person would have to check that the first cell was full and the technique was therefore rather cumbersome.

Ram et al. [1995] subsequently developed the technique of LLS on solid bubble-free ice to measure how the dust concentration varies with depth in bubble-free ice below 1700 m. They compared LLS from solid, bubble-free ice with LLS from corresponding meltwater samples and found that the two covaried and concluded that LLS from solid, bubble-free ice would be able to measure the concentration of dust in the ice. As the ice in this setup wasn’t melted and was moved by a motor driven translation stage the LLS measurements was very fast in comparison with the meltwater LLS.

In 1997 Ram and Koenig published a profile of the GISP2 core from the surface to the silty-ice boundary. The top ∼1800 m was examined by meltwater LLS and the rest by solid ice LLS. Their results were compared to electrical conductivity measurements (ECM) and oxygen isotope profiles and displayed all the rapid climatic oscillations during
3.1. LASER LIGHT SCATTERING

the last glaciation (Dansgaard-Oeschger events) that were seen by the oxygen isotope and ECM records. In the lower part of the core (from a depth of 2250 m) they found a chronology in correspondence with gas-age dating of the ice.

The interrogations by Ram and Illing [1994], Ram et al. [1995], and Ram and Koenig [1997] all showed the scattered intensity vs. depth in relative scattering units. That is, no actual calibration of the scattered light to the amount of dust in the ice had been done. In 2000 Ram et al. published a calibration paper, in which the authors calibrated the previous dust profiles using INAA of aluminum on archived ice cores and found that all major impurities present in the ice covaried linearly with the dust profile. The dust concentration could now, in principle, be estimated by the intensity of the 90° scattered light. The calibration was limited to the depth range 1674-2586 m; the upper limit (at 1674 m) was dictated by the onset of bubble-free ice and the lower limit (at 2586 m) was set by the appearance of microbubbles below this limit. Visual inspection of the deep ice (below 2700 m) showed that numerous microbubbles (diameter < 0.3 mm) had emerged in the deeper ice in the months following core extraction. The microbubbles commonly occurred as cloudy bands throughout the deep ice. Similar microbubbles (0.02 mm diameter) were observed in an ice sample in the Dye 3 ice core close to the bedrock. These air bubbles were associated with air clathrates and were not visible in the core immediately after core extraction. It is believed that the emergence of microbubbles happen in the first 3-6 months after core extraction, possibly even less than a month. Ram et al. [2000] found that the microbubbles begin to appear in deeper ice than 2586 m (GISP2), hence the lower limit in depth for the calibration of LLS intensity.

Bay et al. [2001] developed an in-situ dust logger based on the design of an ice logger. In the dust logger design a light source (LEDs) shines into the ice surrounding the borehole. Photons scatter off of air bubbles and dust, some are absorbed, and a small fraction are detected in a downward looking PMT, positioned deeper in the hole, beneath the LEDs. This technique has the advantage that it works both in the upper bubbly ice and the deeper bubble-free ice. In the upper bubbly ice, where scattering occurs predominantly by bubbles, the absorption from dust impurities causes the signal to drop, whereas in the lower bubble-free ice the scattering from dust increases light collection. The drawback of this method is that the resolution is distinctly coarse compared to LLS techniques.

Ruth et al. [2003] developed a laser based particle detector working on meltwater samples. The particle detector is based on laser light attenuation by single particles. The sample meltwater is pumped through an illuminated detection cell, where each particle is detected as a negative peak in transmitted light. In this technique both geometric shadowing and scattering processes are in play and the calibration of particle size from peak height is complex. The great advantage of this setup is that it is capable of measuring
both particle concentration and size distribution of the insoluble particles in the melted ice sample. Ruth et al. [2003] made a continuous record of insoluble particle concentration and size distribution of 1500 m of the NGRIP ice core. The particle concentration had a depth resolution of about 1 cm and the size distribution had a depth resolution of 1.7 m.

3.2 Instrument Geometry

The central objective of this work is a design and feasibility study for an optical, single or multi-wavelength Mie-scatterometer to be integrated into the JPL Cryobot deployed into an ice sheet; as a consequence the geometry of the instrument is constrained in many ways. This includes the need for the camera and laser to be in close proximity to each other in the hull of the Cryobot. A setup where the laser had a perpendicular incidence with respect to the optical axis of the camera would provide a data stream with a stronger linearity with respect to particle density, as in most laboratory setups [Ram and Koenig, 1997; Ram et al., 2000], but that geometry is not applicable in our situation. Below is a description of the idea behind the instrument and a detailed discussion of the optimization of the design of the Laser Dust Detector in the presence of these geometrical constraints.

3.2.1 The Field Deployed Instrument

Figure 3.1 shows the geometry of the Laser Dust Detector in the field deployed configuration (i.e. integrated into the Cryobot). The data acquired by the instrument consists of a series of digital images, taken by the camera, of laser light scattered by objects in the ice. Laser light is directed via fiber optics into the meltwater and ice surrounding the probe. If the ice is bubble and dust free the laser light will penetrate meters of ice before the light is attenuated completely (see chapter 4 for a discussion of attenuation). If, however, the laser light encounters dust particles the light will be scattered in all directions, according to the discussion of Rayleigh and Mie scattering in chapter 4. Some of the scattered light will make its way into the aperture of the CCD camera and be detected in a digital image. A series of these images are taken as the Cryobot descends into the ice sheet. The images will be acquired in a way to ensure enough overlap between neighboring images to be able to fit them together to get a continuous in-situ mapping of the dust in this local environment of the ice sheet.

Figure 3.1 shows that, in the Cryobot geometry, a laser mounted in the hull enables us to detect scattering angles close to 90° with a wide-angle objective lens oriented normal to the Cryobot hull. Another way to get close to a scattering angle of 90° would be

\footnote{The assumption of bubble free ice is valid in the Central Greenland ice sheet below a depth of approximately 1200 m}
3.2. INSTRUMENT GEOMETRY

Figure 3.1: Field deployed geometry. The Laser Dust Detector can be seen as the integrated instrument consisting of the laser and the camera system. A cross section of the camera field of view is shown.

to rotate the camera toward the laser. This, however, is not a good solution in most practical situations as the layering of the dust usually is horizontal, or close to horizontal. A rotation of the camera will thus make individual layers become less defined. Imagine looking through a deck of cards with the individual cards spaced a few mm apart. If you tilt the deck you no longer will be able to see through the deck. In the same way will dust layers 'shadow' each other if the camera is tilted, and the individual layers will become less defined in the images.

3.2.2 The Laboratory Instrument

The data acquired for this work have all been obtained in a laboratory; that is, none of the data were taken with the Cryobot (and integrated Laser Dust Detector) deployed in the field, due to the great cost in deploying the Cryobot. As noted in the introduction, the Cryobot has been field deployed at a glacier on the island of Svalbard, Norway, but the Laser Dust Detector was, at this point in time, not in a state ready for such a deployment.

The main difference in the geometrical setup between the field deployed instrument and the laboratory instrument is that in the laboratory ice cores are being examined, while
in the field it is a local environment of a whole ice sheet being examined. The laboratory setup is shown in Figure 3.2 When deploying the instrument in the field the laser is for all practical purposes shining into an infinitely large piece of ice and will therefore be able to trace out a very long path in the ice. The laser beam will not encounter any abrupt changes in index of refraction (other than that of dust assuming bubble free ice) and will therefore not change its direction significantly before it is completely attenuated. In the laboratory however, the beam will be diffusely reflected when it hits the back wall of the ice core due to the large difference in index of refraction between the ice and the surrounding air. The reflection is mostly diffuse as the back wall of the ice is very rough on the order of the laser wavelength and contains many scattering centers. A fraction of the internally reflected light will therefore be directed toward the camera and induce a large amount of noise in the images. This noise has proved to be a significant problem to overcome but a problem that is ”artificial” in the sense that it will not be present in the case of a field deployment, but nonetheless a problem when analyzing archived ice core data.

Figure 3.2: The geometry of the laboratory setup.
3.2.3 Constraints on Instrument Geometry

When optimizing the geometry and performance of the Laser Dust Detector we will meet certain geometrical constraints and we will encounter a number of near-optimal configurations to choose from. Below is a discussion of these constraints and how to optimize the instrument geometry accordingly.

The derivations are done on the field deployed instrument as this is the natural starting point in designing the instrument. Assume, for the derivations, the following parameters [Shackelford and Alexander, 2000]:

- Real refractive index of fiber optics glass, $n'_g = 1.40$
- Real refractive index of water, $n'_w = 1.33$
- Real refractive index of ice, $n'_{ice} = 1.31$
- Real refractive index of quartz glass, $n'_q = 1.54$
- Angular field of view of the CCD camera, $FOV = 31.3^\circ$

The laser light is directed via an optical fiber through the hull and into the melt water and ice surrounding the probe. When optimizing the geometry we want to maximize the amount of scattered light that will be detected by the camera under the given geometrical constraints. The amount of scattered light that reaches the sensitive area of the camera is in an intricate way dependent on the size and composition of the scatterers, the wavelength of the laser light, and the usable scattering angles. These usable scattering angles are dependent on the incidence angles of the laser light going through the different media from the optical fiber, through the meltwater envelope, into the ice, and back to the detector going through the meltwater and the quartz window in the hull.

By taking into account the refraction of the incident laser beam at the different interfaces (fiber optics glass $\rightarrow$ water and water $\rightarrow$ ice) not all incident laser angles are allowed due to the fact that total reflection at these interfaces must be avoided, see Figure 3.3. Total reflection at any of the interfaces will prevent any light from entering the ice and thereby prevent any data from being collected. Figure 3.3 shows that an incidence angle $\theta_{gw} = 69.3^\circ$ results in total reflection at the water$\rightarrow$ice interface.

A sketch of the field deployed geometry is shown in Figure 3.4. Due to the refraction of light at the interfaces an effective field of view (EFOV) of the camera exists that is numerically smaller than the field of view in a homogeneous medium. The effective field of view is via Snell’s Law found to be $23.8^\circ$, see Appendix C. The effective field of view will be used in the following calculations. As evident from Figure 3.4 a continuous range in
scattering angles, from $\psi_1$ to $\psi_2$, will be detected by the camera assuming a homogeneous distribution of scatterers in the ice. For every laser incidence angle a unique range in corresponding scattering angles exists. In optimizing the performance of the Laser Dust Detector we want to find the range in scattering angles that returns the most scattered light to the detector, under the important constraint that the intensity of the scattered light is fairly uniform over the range in scattering angles.

A number of geometrical parameters can be varied which in turn will change the laser’s incidence angle into the meltwater and thereby change the corresponding range in scattering angles $[\psi_1, \psi_2]$. In the following this range in scattering angles will be determined where it is assumed that the power control of the Cryobot is run close to optimal performance, i.e. only a thin meltwater envelope is maintained around the probe (here it is assumed that a 5 mm thick meltwater envelope exists).

Derivation of Range in Scattering Angles $[\psi_1, \psi_2]$

Starting with the last interface (water $\rightarrow$ ice) the laser encounters a change in index of refraction from $n_w' = 1.33$ to $n_{ice}' = 1.31$. This is a negative change and the possibility

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**Figure 3.3:** An illustration of Snell’s Law. Due to the fact that total reflection must be avoided at all the interfaces constraints on the incident laser angle $\theta_{gw}'$ exist. See Figure 3.5 for a definition of angles.

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<td>69.3°</td>
<td>80.1°</td>
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of total reflection thus exists. At the water-ice interface the laser beam will be totally reflected at incidence angles above a critical angle $\theta_{wi}^i$ of

$$
\theta_{wi}^i = \arcsin \left( \frac{n_{\text{ice}}}{n_w} \sin \theta_{gw}^t \right) = 80.1^\circ,
$$

i.e. the maximum incidence angle for the water-ice interface must be smaller than $80.1^\circ$ to avoid total reflection at this interface (see Figure 3.5 for a definition of the angles). The incidence angle for the water-ice interface $\theta_{wi}^i$ is the same as the transmittance angle for the glass-water interface $\theta_{gw}^t$, thus

$$
\theta_{wi}^i = \theta_{gw}^t.
$$

The subscripts of $\theta_{wi}^i$ and $\theta_{gw}^t$ refer to the water-ice and glass-water interfaces, respectively and the superscripts $i$ and $t$ refer to incidence and transmittance angles, respectively. At the first interface (the fiber optics glass $\rightarrow$ water interface) the maximum transmittance angle into the water must be $\theta_{gw}^t = 80.1^\circ$ (according to 3.1). Using Snell’s Law we find the maximum incidence angle $\theta_{gw}^i$ for the glass-water interface, to be

$$
\theta_{gw}^i = \arcsin \left( \frac{n_w}{n_g} \sin \theta_{gw}^t \right) = 69.3^\circ.
$$
Thus, by changing the angle of the laser beam in the hull of the Cryobot, from 0° to 69.3° with respect to a normal to the Cryobot hull, the angle inside the ice changes from 0° to 90°.

Having found the upper limit $\theta_{gw} = 69.3^\circ$ on the incidence angle of the laser beam in the hull, we now seek to find the lower limit. We want the laser beam to cross the entire field of view of the camera without penetrating too far into the ice, see Figure 3.4. As the beam crosses the field of view of the camera it has the potential of being scattered, resulting in attenuation of the beam. Because of this attenuation of the laser beam, we introduce a maximum distance into the ice that we want scattering, and subsequent detection, to take place. This distance will be defined as the maximum distance in the detection range. In Figure 3.4 the point $q$ defines the point where the laser beam exits the field of view at the maximum detection distance. The point $p$, where the laser beam enters the field of view, defines the minimum distance where we want scattering and subsequent detection to take place. The detection distances are measured with respect to the edge of the ice, and the minimum detection distance is desirably not zero (i.e. at the edge of the ice itself). If excessive heating is taking place, cracks and fissures can be introduced into the ice which could refract and/or scatter the laser beam undesirably. The point $p$ together with point $q$ defines the lower limit on the incidence angle $\theta_{gw}$ of the laser beam in the hull, see Figure 3.5.

The object of the current calculations is to find all the possible geometrical configurations for the Laser Dust Detector and compare these to find the geometry that enables the camera to detect the highest amount of scattered light. By changing the laser incidence angle in the hull from the lowest permissible angle (dependent on the minimum and
3.2. INSTRUMENT GEOMETRY

maximum detection distances) to the highest (defined by the need to avoid total reflection at all interfaces) all the different geometrical configurations of the Laser Dust Detector are found. Empirically derived good values for the minimum and maximum detection distances\(^2\) for Greenland ice are

Minimum distance = 10 mm and maximum distance = 100 mm.

Given the minimum and maximum detection distances as well as the diameter of the quartz glass window in the hull and the thickness of the meltwater envelope the lower limit of the laser incidence angle can be found. Using realistic values for these parameters (melt water envelope = 5 mm, window diameter = 20 mm, and EFOV = 23.8\(^\circ\)) we first derive the lower limit of the laser transmittance angle into the ice \(\theta_{wi}^t = 26.7^\circ\), see Appendix C for calculation of \(\theta_{wi}^t\).

Calculating backwards, this angle corresponds to a laser angle in the hull (incidence angle into the water) \(\theta_{gw}^i\) of

\[
\theta_{gw}^i = 26.7^\circ \quad \text{derived from empirical values in Appendix C}
\]

\[
\theta_{wi}^i = \arcsin \left( \frac{n'_{i,ce}}{n'_{gw}} \sin \theta_{wi}^t \right) = \theta_{gw}^t
\]

\[
\theta_{gw}^i = \arcsin \left( \frac{n'_{i,w}}{n'_{gw}} \sin \theta_{gw}^t \right) = \arcsin \left( \frac{n'_{i,w}}{n'_{gw}} \sin \left( \arcsin \left( \frac{n'_{i,ce}}{n'_{gw}} \sin \theta_{wi}^t \right) \right) \right)
\]

\[
= \arcsin \left( \frac{n'_{i,ce}}{n'_{gw}} \sin \theta_{wi}^t \right) = 24.9^\circ
\]

Thus, changing the angle of the laser beam in the Cryobot hull, through the permissible angles \(\theta_{gw}^i\), corresponds to the following transmittance angles \(\theta_{wi}^t\) into the ice:

\[
\theta_{wi}^t \in [24.9^\circ, 69.3^\circ] \rightarrow \theta_{wi}^t \in [26.7^\circ, 90.0^\circ],
\]

\(\theta_{gw}^i = 69.3^\circ\) and \(\theta_{wi}^t = 90^\circ\) not included.

To ease the notation, the transmittance angle into the ice \(\theta_{wi}^t\), will in the following be named \(\theta_{ice}^t\) and the incidence angle into the glass-water interface \(\theta_{gw}^i\) will be named \(\theta_{hull}\). The task of optimizing the performance of the Laser Dust Detector now consists of choosing one of all the possible incidence angles \(\theta_{hull} \in [24.9^\circ, 69.3^\circ]\). In fact, what we really want to optimize is the amount of scattered light that is detectable by the camera. The actual dust level in the interior of the Mars ice caps is not known; since we know that Antarctic ice is nearly dust free we need to maximize the sensitivity of the Laser Dust Detector. That is, we want to choose that incidence angle \(\theta_{hull}\) that will result in the highest amount of scattered light reaching the camera.

\(^2\)The values for the detection distances are found by examining archived ice cores from the GISP2 drill site on Greenland. These cores have a diameter of 133 mm.
Referring to the sketch of the geometry, Figure 3.4, the light being scattering with an angle between $\psi_1$ and $\psi_2$ will be detected by the camera, all the rest will be lost. Going through the range of permissible incidence angles $\theta_{hull}$ the corresponding scattering angles $[\psi_1, \psi_2]$ are given as (see Appendix C for a derivation of the scattering angles),

$$\begin{align*}
\psi_1(\theta_{hull}) &\in [78.1^\circ, 141.4^\circ] \\
\psi_2(\theta_{hull}) &\in [101.9^\circ, 165.2^\circ]
\end{align*}$$

, for $\theta_{hull} \in [24.9^\circ, 69.3^\circ]$.

The lower limit for $\psi_1$ and $\psi_2$ are not included as they correspond to the case of total reflection at the water-ice interface. The smaller scattering angles might be permissible from a geometrical point of view but could be impossible to implement from an engineering point of view. For a laser beam, with an incidence angle of $\theta_{hull} = 69.2^\circ$, to cross the field of view at a distance of 10 mm from the water-ice interface, the laser must be positioned 306 mm below the optical axis of the camera system. This is a long distance in the small compartment of the Pressure Vessel. Thus the mechanical engineering of the probe poses other constraints on the geometry. The largest incidence angles will therefore be avoided, although they are permissible in a strict sense.

Figure 3.6 shows the two scattering angles ($\psi_1$ and $\psi_2$) plotted as a function of $\theta_{hull}$.

![Figure 3.6](image)

Figure 3.6: For every possible laser incidence angle $\theta_{hull}$, there exists a continuum of scattering angles, limited by $\psi_1$ and $\psi_2$. For one incidence angle the difference between $\psi_1$ and $\psi_2$ corresponds to the angular field of view of the camera. For $\theta_{hull} = 60.0^\circ$ the corresponding scattering angles lie in the range $[100.4^\circ, 124.2^\circ]$ marked by the dashed lines.

Thus, if one was to choose a geometry that has an angle $\theta_{hull}$ of, say $60.0^\circ$, the corresponding range in scattering angle would be between $\psi_1 = 100.4^\circ$ and $\psi_2 = 124.2^\circ$, see Figure 3.6. Note that one always has $|\psi_2 - \psi_1| = EFOV = 23.8^\circ$. 


3.2. INSTRUMENT GEOMETRY

When choosing the geometry for the Laser Dust Detector we would thus pick a setup where the 23.8° of angular field of view collects the largest intensity of the scattered light. This brings us to a discussion of the intensity of light scattered from a spherical particle in the ice. A thorough discussion of light scattered by small particles follows in the next chapter.
Chapter 4

Scattering of Light by Small Particles

4.1 Introductory Electromagnetic Theory

We can acquire a qualitative understanding of the physics of light scattering by a single particle by considering an arbitrary particle, which we imagine is subdivided into small regions. An applied oscillating electromagnetic field induces a dipole moment in each region. These dipoles oscillate at the same frequency as the applied field and therefore scatter so called secondary radiation in all directions. At a distant point in a particular direction we observe the total scattered field by superposing the scattered wavelets, where their phase differences are taken into account. We generally expect the scattered field to vary with the scattering direction as the phase relations change for a different scattering direction. If the particle is small compared with the wavelength of the applied field, all the secondary wavelets are approximately in phase. For such a particle we do not expect much variation of scattering with direction. This type of scattering is in the domain governed by what is called Rayleigh scattering. As the particle size is increased chances of mutual constructive and destructive interference of the scattered wavelets increases. Thus, the larger the particle is, the more structure (peaks and valleys) we expect in the scattering pattern. At a certain point in particle size, depending on the wavelength of the applied field, we enter the domain of Mie scattering. Figure 4.1 schematically shows the difference in the pattern of scattered light for the two domains.

The theories for Rayleigh scattering and for the so called geometrical optics (for particles much much larger than the wavelength of the applied field), are in fact both approximations to the full Mie theory. These approximations provide a faster way of solving the scattering problem for much smaller and much larger particles. Today routine computing power is so large that the full Mie solution can be derived in a practical time frame. The
Figure 4.1: The figure schematically shows the difference in scattering pattern for Mie scattering (top) and Rayleigh scattering (bottom). The size parameter $x$ is the ratio of particle circumference to the wavelength of the incident radiation. $a$ is the particle radius. Rayleigh scattering, for small particles, is characterized by a more isotropic distribution of the scattered electromagnetic wave whereas Mie scattering, for larger particles, has a pronounced forward scattering lobe.

Below follows a brief summary of the electromagnetic theory needed for understanding the scattering of electromagnetic radiation by particles. A detailed description with complete derivations can be found in e.g. Bohren and Huffman [1983] and van de Hulst [1957].

4.1.1 Poynting vector

The Poynting vector is of fundamental importance in problems of propagation, absorption and scattering of electromagnetic waves. Consider an electromagnetic field with an electric component $\mathbf{E}$ and a magnetic component $\mathbf{H}$. The *Poynting vector* $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ specifies the magnitude and direction of the rate of transfer of electromagnetic energy at all points in space. The net rate $W$ at which electromagnetic energy is transferred across the boundary of a closed surface $A$ which encloses a volume $V$ is then given as

$$ W = - \int_A \mathbf{S} \cdot \hat{n} \, dA, $$
4.1. INTRODUCTORY ELECTROMAGNETIC THEORY

where \( \hat{n} \) is a unit normal vector to the surface \( A \), and \( \mathbf{S} \) is assumed to be a function of position (i.e. not necessarily constant over the surface). \( \hat{n} \) is chosen to point out of the volume which implies that \( W \) is positive if there is a net transfer of electromagnetic energy into the volume, i.e. a positive \( W \) implies that energy is absorbed in \( V \) (converted into other forms of energy).

In the following it will be the time-averaged Poynting vector that will be used, where plane waves are assumed,

\[
\langle \mathbf{S} \rangle = \frac{1}{2} \text{Re}\{\mathbf{E}_c \times \mathbf{H}_c^*\},
\]

the subscript denoting that \( \mathbf{E} \) and \( \mathbf{H} \) are complex vectors. \( \mathbf{E}_c \) and \( \mathbf{H}_c \) have the form

\[
\mathbf{E}_c = \mathbf{E}_0 \exp(i\mathbf{k} \cdot \mathbf{z} - i\omega t) \quad \text{and} \quad \mathbf{H}_c = \mathbf{H}_0 \exp(i\mathbf{k} \cdot \mathbf{z} - i\omega t),
\]

(4.1)

for a wave traveling in the positive \( z \) direction and where \( \mathbf{E}_0 \) and \( \mathbf{H}_0 \) are constant vectors, compatible with the Maxwell equations. The wave vector \( \mathbf{k} \) may be complex and the magnitude of the wave vector is given by \( k = 2\pi/\lambda \), where \( \lambda \) is the wavelength in the medium. Given the wavelength in vacuum \( \lambda_0 \), we have \( \lambda = \lambda_0/n \), where the complex refractive index \( n = n' + in'' \). The magnitude of the wave vector \( \mathbf{k} \) thus becomes

\[
|\mathbf{k}| = k = \frac{2\pi}{\lambda} = \frac{2\pi n}{\lambda_0} = \frac{2\pi n'}{\lambda_0} + i\frac{2\pi n''}{\lambda_0}.
\]

Substituting this into (4.1) a plane electric wave has the form

\[
\mathbf{E}_c = \mathbf{E}_0 \exp\left(-\frac{2\pi n''z}{\lambda_0}\right) \exp\left(i \frac{2\pi n'z}{\lambda_0} - i\omega t\right).
\]

(4.2)

\( \omega \) is the usual cyclic frequency of the wave, \( t \) the time dependence and \( z \) is in the direction of propagation. The first exponential part on the right side of (4.2) is the amplitude of the electric wave and the second exponential part is the phase of the wave. The imaginary part of the complex refractive index \( n'' \) thus determines the attenuation of the wave as it propagates through the medium. The real part \( n' \) determines the phase velocity \( v = c/n' \). The pair \( n', n'' \) are often referred to as the optical constants, even though they are not constant but strongly dependent on frequency.

The magnitude of the time-averaged Poynting vector is called the irradiance and its dimension is energy per unit area per unit time or equivalent \([\text{W/m}^2]\). In the following, irradiance will be denoted by \( I \). As the wave traverses the medium the irradiance is exponentially attenuated

\[
I = I_0 e^{-\alpha z},
\]

(4.3)

where the absorption coefficient \( \alpha \) (unit \([\text{m}^{-1}]\)) is

\[
\alpha = \frac{4\pi n''}{\lambda_0}.
\]
and $I_0$ is the irradiance at $z = 0$. Table 4.1 lists the complex index of refraction as well as the absorption coefficients for different materials important for this work.

<table>
<thead>
<tr>
<th>Material</th>
<th>Complex Index of Refraction</th>
<th>Absorption Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz (SiO$_2$)†</td>
<td>$n = 1.53 + 1.75 \times 10^{-3}$</td>
<td>$\alpha = 1.5 \times 10^5$ m$^{-1}$</td>
</tr>
<tr>
<td>Magnetite (Fe$_3$O$_4$)†</td>
<td>$n = 1.53 + i1.0 \times 10^{-1}$</td>
<td>$\alpha = 2.0 \times 10^6$ m$^{-1}$</td>
</tr>
<tr>
<td>Pure ice‡</td>
<td>$n = 1.31 + i1.0 \times 10^{-8}$</td>
<td>$\alpha = 0.2$ m$^{-1}$</td>
</tr>
<tr>
<td>Pure water‡</td>
<td>$n = 1.33 + i1.6 \times 10^{-8}$</td>
<td>$\alpha = 0.3$ m$^{-1}$</td>
</tr>
</tbody>
</table>

Table 4.1: Complex index of refraction and absorption coefficients for different materials. Compared to the absorption of visible light in dust particles (e.g. quartz or magnetite) the absorption in water or ice is negligible. † from Royer (1981); ‡ from Mobley (1994).

### 4.1.2 Polarization

In a plane perpendicular to the direction of light propagation the endpoint of a vector representing the electric field at a fixed point in space traces out an ellipse with time. Special cases of this ellipse are a straight line and a circle, corresponding to linear and circular polarization, respectively.

In general light is partially polarized, where the electric vector changes but has a generally preferred orientation. The light is completely polarized only if the orientation and ellipticity of the polarization ellipses are identical for each simple wave. Direct sunlight represents the other extreme. It is a mixture of uncorrelated simple waves, and over a time period for usual measurements the electric vector exhibits no preferred direction of vibration.

An arbitrary beam of light of irradiance $I$ can be separated into an unpolarized part and a totally polarized part,

$$I = I_{unpol} + I_{pol}$$

The degree of polarization is defined as the ratio $I_{pol}/I$. The polarized part of the beam is in general elliptically polarized, and it can be further separated into a linearly polarized part of intensity $I_{lp}$ and a circularly polarized part of intensity $I_{cp}$, where

$$I_{pol} = (I_{lp}^2 + I_{cp}^2)^{1/2}.$$ 

The laser used in the experimental part of this work has a degree of polarization of 100, i.e. the irradiance of the polarized part of the beam is 100 times the irradiance of the unpolarized part of the beam.
4.1.3 Reflection and Transmission at Oblique Incidence

Consider a plane wave propagating in a non-absorbing medium with real refractive index $n_i$, which is incident on an absorbing medium with complex refractive index $n_t = n'_t + i n''_t$. Throughout this work the italic $i$ denotes incidence and the roman $i$ denotes $\sqrt{-1}$, unless otherwise specified. The amplitude of the incidence electric field is $E_i$, and it is assumed that there exists transmitted and reflected waves with amplitudes $E_t$ and $E_r$, respectively. All plane waves normally incident on a plane boundary are reflected and transmitted independently of their state of polarization. However, when a plane wave is obliquely incident on a boundary the polarization of the incident wave is important. Incident, unpolarized light may become highly polarized at certain angles when reflected at a plane boundary or scattered by a particle. Therefore, we consider two polarizations in discussing reflection of light incident at an arbitrary angle on a plane boundary. One polarization has the electric field vector parallel to the plane of incidence, the other has the electric field vector perpendicular to the plane of incidence, see Figure 4.2. The plane of incidence is defined by the direction of propagation of the incident wave and a normal to the boundary. An arbitrary wave may be written as a superposition of these two polarizations. Also, the two polarizations are independent of each other. If the incident wave has a certain polarization, say polarized parallel to the plane of incidence, the reflected and transmitted wave have the same polarization.

When light is incident upon a medium of larger index of refraction $n_t > n_i$, the transmitted ray is bent toward the normal, and the transmittance angle $\theta_t$ will be smaller than the incident angle $\theta_i$. This is the case seen in the schematics of Figure 4.2. On the other hand if the incident medium has a smaller index of refraction the transmitted ray will be bent away from the normal. This is commonly called "internal reflection". The transmittance angle will then approach $90^\circ$ for some critical incident angle $\theta_c$, and for incident angles greater than the critical angle there will be total internal reflection. From Snell’s law the critical angle can be found by

$$\sin \theta_c = \frac{n_t}{n_i}.$$ 

As an example we look at a beam of laser light inside a piece of ice surrounded by air. The beam will exit the ice if the incidence angle to the ice-air interface is smaller than $\theta_c = \arcsin \left( \frac{n_{\text{air}}}{n_{\text{ice}}} \right) = \arcsin \left( \frac{1.00}{1.31} \right) = 49.8^\circ$.

Exceeding this angle will prevent the light beam from exiting the ice at this interface.

The Fresnel equations relate the amplitudes of the reflected wave $E_r$ or transmitted wave $E_t$ to that of the incident electric wave $E_i$:

$$E_{\perp r} = r_{\perp} E_{\perp i}$$
Figure 4.2: Oblique incidence on a plane surface. In the top schematics the electric field vector is parallel to the plane of incidence (equal to the plane of the paper). The bottom figure has the electric field vector perpendicular to the plane of incidence.

\[
E_{\perp t} = t_{\perp} E_{\perp i},
\]
\[
E_{\parallel r} = r_{\parallel} E_{\parallel i},
\]
\[
E_{\parallel t} = t_{\parallel} E_{\parallel i},
\]

where the so called reflection coefficients \( r_{\perp} \), \( r_{\parallel} \) and transmission coefficients \( t_{\perp}, t_{\parallel} \) for perpendicular and parallel polarized incident light are given as

\[
r_{\perp} = \frac{-\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \quad (4.4)
\]
\[
t_{\perp} = \frac{2 \cos \theta_i \sin \theta_t}{\sin(\theta_i + \theta_t)} \quad (4.5)
\]
\[
r_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \quad (4.6)
\]
\[
t_{\parallel} = \frac{2 \cos \theta_i \sin \theta_t}{\sin(\theta_i + \theta_t) \cos(\theta_i - \theta_t)} \quad (4.7)
\]

Here it is assumed that the permeabilities of the dielectric materials are close to that of vacuum. The reflectance and transmittance are defined as the ratio of reflected and transmitted irradiance, respectively, to the incident irradiance. Figure 4.3 shows plots of
the reflectance $R$ and transmittance $T$ given as

$$
R_{\perp} = r_{\perp}^2 \\
T_{\perp} = \left(\frac{n_t \cos \theta_t}{n_i \cos \theta_i}\right) t_{\perp}^2 \\
R_{\parallel} = r_{\parallel}^2 \\
T_{\parallel} = \left(\frac{n_t \cos \theta_t}{n_i \cos \theta_i}\right) t_{\parallel}^2.
$$

Furthermore we have $R_{\text{unpol}} = 1/2(R_{\perp} + R_{\parallel})$ and $T_{\text{unpol}} = 1/2(T_{\perp} + T_{\parallel})$ for unpolarized incident light. The polarization angle $\theta_p$ is defined as the incidence angle for which $R_{\parallel} = 0$. If the incident light is unpolarized the reflected light will be completely polarized perpendicular to the incidence plane at this angle. The polarization angle is given as

$$
\tan \theta_p = \frac{n_t}{n_i}.
$$

In Figure 4.3 (right) the reflectance and transmittance of the laser beam are plotted as functions of the incidence angle in the case of the Cryobot geometry, i.e. when the 100:1 polarized laser beam penetrates two interfaces. $T_{gw}$ and $R_{gw}$ are the transmittance and reflectance for the fiber optics glass→water interface, respectively. The incidence angle is limited to the range $[0, 69.3^\circ]$ to avoid total reflection at the water→ice interface. $T_{wi}$ and $R_{wi}$ are the transmittance and reflectance at the water→ice interface. The incidence angle at this interface is given as the transmission angle for the glass→water interface, i.e. at $69.3^\circ$ the incidence angle at the glass→water interface is equal to $90.0^\circ$, and the curve for $R_{wi}$ shows total reflection. $T_{gi}$ and $R_{gi}$ are the transmittance and reflectance for the combined interfaces, going from the fiber optics glass, through the meltwater envelope and
into the ice. These are the resulting curves for the laser beam in the Cryobot geometry
and at an incidence angle of, say 60° the transmittance equals $T_{gi} = 0.98$, showing that
only a small fraction of incident light is reflected at the two interfaces.

As mentioned, the laser used throughout the experimental part of this work has a
polarization of 100:1. Plotting the reflectance and transmittance of a laser with a polar-
ization of 100:1 yields curves that are indistinguishable from the ones in Figure 4.3. This
means that our laser for all practical purposes is 100% polarized. In the case of a laser
beam incident on a piece of ice surrounded by air the polarization angle is $\theta_p = 52.6^\circ$.
This has implications when one tries to minimize the amount of light reflected on the
surface of the ice. If one wants to interrogate the interior of the ice with a laser beam and
a camera, any reflections on the surface of the ice have the possibility of finding the way
back to the camera and thus adding noise to the image. In theory, this can be avoided
by choosing the incidence angle equal to the polarization angle, thereby removing the
parallel polarized part of the beam. Only the perpendicularly polarized part of the beam
remains which can be blocked by a polarization filter in front of the camera.

4.2 Mie Scattering

Let us consider the following problem. We have a single homogenous and spherical particle
of radius $a$, whose material has a complex index of refraction $n_p = n'_p + in''_p$. The sphere
is embedded in a uniform, non-absorbing medium with index of refraction $n_m = n'_m$
($n_m$ is real as the medium is non-absorbing). The sphere is illuminated by a beam of
monochromatic light with wavelength $\lambda = \lambda_0/n'_m$, where $\lambda_0$ is the wavelength of the light
in vacuum. We wish to find how the incident light is absorbed and scattered by the
particle, including the angular distribution of the scattered irradiance and the state of
polarization of the scattered light. *Mie theory* gives a rigorous solution to the problem.

The beam of light is described as an electromagnetic plane wave and the sphere is
treated as a dielectric. Maxwell’s equations are solved subject to boundary condition
at the surface of the sphere. The solution to Maxwell’s equations gives the electromagnetic
field both within the sphere and in the surrounding medium. Gustav Mie solved
this problem in 1908 [Mie, 1908]. His general solution is exact and valid for all sizes of
spheres, refractive indices, and wavelengths. The solution is in the form of infinite series
of mathematical functions (Ricatti-Bessel and Legendre functions). A complete mathema-
tical derivation of the solution will not be stated here; details can be found in Bohren
and Huffman [1983] and van de Hulst [1957]. We will however familiarize ourselves with
the terminology and give a physical interpretation of some of the various parameters
encountered.
4.2. MIE SCATTERING

The Mie solutions can be derived as functions of two important parameters, the size parameter $x$ and the relative refractive index $m$. It is an inherent nature of Mie scattering that the same scattering patterns are obtained illuminating a 1 mm radius airborne ice crystal with a microwave beam or illuminating a 1 micron radius dust particle embedded in ice with a visible laser beam. The important parameters are the relative refractive index $m$ and the size parameter $x$ defined as

$$x = \frac{2\pi a}{\lambda}, \text{ where } \lambda = \frac{\lambda_0}{n_m}$$

$$m = \frac{n_p}{n_m} = \frac{n'_p + in''_p}{n'_m},$$

where $\lambda_0$ is the wavelength in vacuum and $a$ the radius of the particle. The importance of the size parameter is sketched in Figure 4.1.

Mie’s solutions are often described in term of scattering and absorption cross sections or efficiencies. If we define the ”energy incident on a sphere” by the energy passing through an area equal to the cross-sectional area of a sphere (i.e. the projected or ‘shadowed’ area $A_s = \pi a^2$), then the scattering efficiency $Q_{sca}$ is defined as the ratio of energy scattered by the sphere to the energy incident on the sphere. Similarly the absorption efficiency $Q_{abs}$ is the ratio of energy absorbed by the sphere to the energy incident on the sphere. The extinction efficiency $Q_{ext}$ is then given as $Q_{ext} = Q_{sca} + Q_{abs}$. The efficiencies are dimensionless. In Figure 4.4 the Mie solution for the efficiencies for scattering, absorption, and extinction are plotted as functions of the size parameter. Note that the extinction efficiency for a non-absorbing sphere approaches 2. This means that twice the amount of energy incident on the particle is removed from the incident beam, something that apparently contradicts ’common sense’ as one does not expect a large object to remove twice the energy that is incident on it. This is therefore also termed the extinction paradox. First of all, an amount of energy $I_i \pi a^2$ is removed from the beam with irradiance $I_i$ as a result of reflection, refraction and absorption of the rays that are incident on the sphere. This gives us the first part of the extinction efficiency. Every ray that is either absorbed or changes its direction is counted as having been removed from the incident beam. The ‘solution’ to the paradox comes from the notion of diffraction. An equal amount of energy is scattered or diffracted by the particle. Roughly speaking, we may say that the incident wave is influenced beyond the physical boundaries of the particle. The edge deflects rays in its neighborhood that, in the ’classical sense’, would have passed unhindered. Adding the two contributions gives the total of twice the amount of light incident on the particle.

For a non-absorbing particle the absorption efficiency is zero and therefore the extinction efficiency (the black curve in Figure 4.4) equals the scattering efficiency. The maxima and minima are due to interference of light diffracted and transmitted by the
Figure 4.4: A plot of the scattering, absorption, and extinction efficiencies $Q_{sca}$, $Q_{abs}$ and $Q_{ext}$, respectively, as functions of the size parameter $2\pi a/\lambda$. Note that for a non-absorbing sphere the extinction efficiency approaches 2. The black curve represents a non-absorbing particle; the red curves an absorbing particle.

A light ray passing through the diameter of the particle is phase shifted and interferes with a diffracted light ray which is not phase shifted. This is the reason for the undulating nature of the curve. For an absorbing particle (the red curves) the scattering efficiency has a maximum (at $x = 15$) and decreases hereafter. The absorption, on the other hand, increases for larger particles and the combined effect is that the extinction efficiency approaches 2 asymptotically for very large particles, as for a non-absorbing particle. From (4.3) the irradiance of a light wave traversing the diameter of a spherical particle is decreased by

$$\frac{I}{I_0} = \exp\left(-\frac{4\pi n'' 2a}{\lambda_0}\right),$$

where the distance travelled is $z = 2a$ and the refractive index is that of the particle. Substituting with the size parameter (4.8), the irradiance is decreased by

$$\frac{I}{I_0} = \exp(-4x). \quad (4.10)$$

Thus the curve for the scattering efficiency will still undulate but will decrease overall according to (4.10).
4.2. MIE SCATTERING

The Mie solution can also be presented in terms of scattering and absorption cross sections. The physical meaning of the scattering cross section $C_{sca}$ is: the cross sectional area of an incident beam that has the same power as the power scattered by the particle. The cross sections have the dimension of area and are related to the efficiencies by the geometrical cross section $A_s$ of the sphere ($A_s = \pi a^2$),

\[
\begin{align*}
C_{sca} &= A_s Q_{sca} = \pi a^2 Q_{sca} \\
C_{abs} &= A_s Q_{abs} \\
C_{ext} &= A_s Q_{ext} = \pi a^2 (Q_{sca} + Q_{abs}).
\end{align*}
\]

Before we go into the derivations of Mie theory a few parameters of light scattering will be discussed. The differential scattering cross section $dC_{sca}/d\Omega$, is defined as the energy scattered per unit time into a unit solid angle about a direction $\Omega$, for unit incident irradiance. The differential scattering cross section is expressed in term of the scattered irradiance $I_s$, the incident irradiance $I_i$, and the distance to the detector $r$ as

\[
\frac{dC_{sca}}{d\Omega} = \frac{r^2 I_s}{I_i}.
\]

The normalized phase function $p(\theta)$ is introduced as

\[
p(\theta) = \frac{1}{C_{sca}} \frac{dC_{sca}}{d\Omega} = \frac{1}{C_{sca}} \frac{r^2 I_s}{I_i},
\]

i.e. the phase function is given as the differential scattering cross section divided by the total scattering cross section, and physically describes what fraction of the scattered irradiance appears per unit solid angle in the direction $\theta$. The word phase has come from astronomy (lunar phases) and has nothing to do with the phase of a wave. (4.11) agrees with that of Bohren and Huffman [1983] and van de Hulst [1957] but differs by a factor of $4\pi$ from phase functions of other authors e.g. Hansen and Travis [1974]. The phase function is normalized so the integration of $p$ over $4\pi$ is equal to unity, i.e.

\[
\int_{4\pi} p d\Omega = 1.
\]

The phase function $p$ governs the scattering of unpolarized light in any direction. Actually $p$ is the element in the first row and first column, $P_{11}$, of the phase matrix $P$, a $4 \times 4$ matrix governing the angular distribution and polarization of scattered light, for any polarization of incident light. The shape of the phase function can usefully be characterized by a single number, called the asymmetry parameter $g$. It is the average or statistically expected value of the cosine of the scattering angle for the scattered light, and therefore often denoted $\langle \cos \theta \rangle$,

\[
g \equiv \langle \cos \theta \rangle \equiv \int_{4\pi} p \cos \theta d\Omega.
\]
For a particle that scatters light isotropically $<\cos \theta>$ is equal to zero. $<\cos \theta>$ also vanishes if the scattering is symmetric about a scattering angle of $90^\circ$. If the particle scatters more light toward the forward direction (the scattering angle $\theta$ approaching $0^\circ$), $<\cos \theta>$ is positive, and $<\cos \theta>$ is negative if the scattering is directed more toward the back-scattering direction ($\theta$ approaching $180^\circ$). Figure 4.5 shows $<\cos \theta>$ as a function of the size parameter. For very small particles, the size parameter $x \to 0$ and $<\cos \theta>$ will approach 0. A light ray passing through a very small particle ($x \to 0$) will have a negligible phase shift and will therefore not interfere constructively and destructively with a diffracted ray. This way the scattered wave will not show any ‘preferred direction’ of scattering and the asymmetry parameter will be very small ($<\cos \theta> \to 0$). As $x$ becomes large, for large particles, a forward scattering lobe will dominate and $<\cos \theta>$ will approach 1. The large maxima and minima have the same physical origin as those in $Q_{sca}$ discussed above. The small ripples superposed on the large undulation arise from so called edge rays, i.e. from light rays grazing the sphere, (see Hansen and Travis [1974] for a detailed discussion of edge rays). In the case of the absorbing particle only the major undulations persist. The small ripples are gone.

The process of finding solutions to the Mie theory is to obtain three-dimensional solutions for Maxwell’s equations for the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{H}$, respectively. The solutions should be found inside and outside a spherical region of radius $a$ (the radius of the particle), taking into account the wavelength of the wave and refractive indices of the particle and the medium into which it is embedded. The task is one of finding a set of complex numbers $a_l$ and $b_l$ (the Mie coefficients) which give vectors $\mathbf{E}$ and $\mathbf{H}$ that satisfy the boundary conditions at the surface of the sphere (at $r = a$).
4.2. MIE SCATTERING

The scattering process is described fully by two complex functions \( S_1(\theta, a_l) \) and \( S_2(\theta, b_l) \) which give the complex amplitude of the scattered wave in terms of the incident complex amplitudes. If \( E_{||i} \) and \( E_{\perp i} \) are the complex amplitudes of the incident electric field resolved along two mutually perpendicular directions, then the corresponding amplitudes of the scattered wave are

\[
\begin{pmatrix}
E_{||s} \\
E_{\perp s}
\end{pmatrix} = \frac{e^{ik(r-z)}}{-ikr} \begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix} \begin{pmatrix}
E_{||i} \\
E_{\perp i}
\end{pmatrix}
\] (4.12)

The explicit formulae for \( S_1 \) and \( S_2 \) are

\[
S_1 = \sum_{l=1}^{\infty} \frac{2l+1}{l(l+1)} \left[ a_l \pi_l(\cos \theta) + b_l \tau_l(\cos \theta) \right] (4.13)
\]

\[
S_2 = \sum_{l=1}^{\infty} \frac{2l+1}{l(l+1)} \left[ b_l \pi_l(\cos \theta) + a_l \tau_l(\cos \theta) \right] (4.14)
\]

where the Mie coefficients \( a_l \) and \( b_l \) are given as functions of the Riccati-Bessel functions \( \psi_l(\rho) \) and \( \xi_l(\rho) \)

\[
a_l = \frac{m \psi_l(mx) \psi'_l(x) - \psi_l(x) \psi'_l(mx)}{m \psi_l(mx) \xi'_l(x) - \xi_l(x) \psi'_l(mx)} (4.15)
\]

\[
b_l = \frac{\psi_l(mx) \psi'_l(x) - m \psi_l(x) \psi'_l(mx)}{\psi_l(mx) \xi'_l(x) - m \xi_l(x) \psi'_l(mx)} (4.16)
\]

\( m \) is the ratio of the refractive index for the absorbing particle and the non-absorbing medium. These complex valued coefficients provide the full solution to the scattering problem. As the refractive index \( m \) approaches unity the coefficients \( a_l \) and \( b_l \) vanish. This is expected; if the particle disappears \((m \to 1)\) so does the scattered field.

\( \pi_l(\cos \theta) \) and \( \tau_l(\cos \theta) \) are functions of the scattering angle. They are related to the Legendre polynomials and are computed from recursion relations. The first two are

\[
\pi_1(\cos \theta) = 1 \quad \tau_1(\cos \theta) = \cos \theta \\
\pi_2(\cos \theta) = 3 \cos \theta \quad \tau_2(\cos \theta) = 3 \cos 2\theta.
\]

Any difference in the phase angle between the complex numbers \( S_1 \) and \( S_2 \) represents a phase difference between the \( || \) and \( \perp \) polarizations in the scattered field. If one is interested in measuring the irradiance, phase differences are irrelevant as the scattered irradiances for the two polarizations are given by the square of the modulus of the complex scattering functions \( |S_1|^2 \) and \( |S_2|^2 \). Generally, if polarization is of concern the Stokes parameters \((I, Q, U, V)\) (see e.g. Bohren and Huffman [1983] for a discussion of the Stokes parameters) can be used to get the full description of the polarization of the scattered
CHAPTER 4. SCATTERING OF LIGHT BY SMALL PARTICLES

radiation

\[
\begin{pmatrix}
I_s \\
Q_s \\
U_s \\
V_s
\end{pmatrix} = \frac{1}{k^2r^2}
\begin{pmatrix}
S_{11} & S_{12} & 0 & 0 \\
S_{12} & S_{11} & 0 & 0 \\
0 & 0 & S_{33} & S_{34} \\
0 & 0 & -S_{34} & S_{33}
\end{pmatrix}
\begin{pmatrix}
I_i \\
Q_i \\
U_i \\
V_i
\end{pmatrix},
\]

(4.18)

\[
S_{11} = \frac{1}{2}(|S_1|^2 + |S_2|^2), \quad S_{12} = \frac{1}{2}(|S_2|^2 - |S_1|^2)
\]

\[
S_{33} = \frac{1}{2}(S_1S_2^* + S_2S_1^*), \quad S_{34} = \frac{i}{2}(S_1S_2^* - S_2S_1^*)
\]

(4.19)

If we denote by \(i_\parallel\) and \(i_\perp\) the scattered irradiance per unit incident irradiance given that the incident light is completely polarized parallel and perpendicular, respectively, to the scattering plane, then

\[
\parallel: \quad i_\parallel = \frac{1}{k^2r^2}(S_{11} + S_{12}) = \frac{1}{k^2r^2}|S_2|^2
\]

(4.20)

\[
\perp: \quad i_\perp = \frac{1}{k^2r^2}(S_{11} - S_{12}) = \frac{1}{k^2r^2}|S_1|^2
\]

(4.21)

unpolarized: \(i_{\text{unpol.}} = \frac{1}{k^2r^2}S_{11} = \frac{1}{2}(i_\perp + i_\parallel)\).

(4.22)

Figure 4.6 – 4.8 show polar plots of \(i_\perp\), \(i_\parallel\), and \(i_{\text{unpol.}}\) for different size parameters. All the plots have been calculated for a non-absorbing spherical dust particle \((n_p = 1.53 + i0)\) embedded in non-absorbing ice \((n_m = 1.31)\), and a wavelength of 632 nm. In Figure 4.6,
4.2. MIE SCATTERING

Figure 4.7: As above but for particle radius \( a = 0.1 \, \mu m \) (x=1).

Figure 4.8: As above but for particle radius \( a = 1.0 \, \mu m \); (x=10). The scattering pattern is characteristic of Mie scattering. The black curve shows the true scale of the scattering pattern; the red curve is a 110 times magnification of the curve to illustrate the undulatory nature of the pattern for \( \theta > \pm 40^\circ \).

\( i_\parallel \) is symmetric around 90°. This type of scattering is characteristic of Rayleigh scattering where \( x \ll 1 \). The plot in Figure 4.7 is for a particle with radius \( a = 0.1 \, \mu m \) (\( x = 1 \)). Although this still is a small particle the scattered radiation is no longer completely isotropic. A forward scattering lobe is starting to build. The plot in Figure 4.8 (left) shows \( i_{unpol} \) in the center (black curve) and an enlarged view to emphasize the shape for \( \theta > \pm 40^\circ \) (red curve). The particle radius is \( a = 1.0 \, \mu m \). Here the forward scattering lobe is pronounced which is characteristic for Mie scattering.
4.2.1 Summary of Mie Scattering

A brief summary of Mie scattering is given here of a more descriptive than mathematical nature. The most notable feature of Mie scattering is the forward lobe. For all but the smallest particles a large fraction of energy is scattered in directions close to that of the incident radiation. This is partly due to diffraction of photons which have actually not passed through the particle but only had their initial direction changed a small amount. If one were to start with a very small scatterer (i.e. $x \ll 1$) the scattering diagram would be found to be symmetric, equal amounts of energy being scattered in backward and forward directions, see Figure 4.6 (left). If the particle size is increased the smooth shape of the Rayleigh scattering pattern would first remain, but an asymmetry would begin to develop, implying more forward-directed than backward-directed scattering. When $x$ exceeds 1, the scattering pattern begins to develop peaks and troughs, with roughly the same amounts of peaks and troughs between 0° and 180° as the value of $x$. As $x$ approaches 10, a finer structure begins to develop, which becomes highly oscillatory for $x \gg 1$. This is not seen in Figure 4.8, though, as $x$ is only 10. It can be seen as the ripple structure in $Q_{ext}$ for larger particles, see Figure 4.4 and for the asymmetry factor $<\cos \theta>$ in Figure 4.5.

4.2.2 Approximations to the Mie Theory

Mie theory is the full solution for scattering of electromagnetic radiation on spherical particles of any size and refractive index. It is however quite cumbersome and, especially before the introduction of the personal computer, approximation to the Mie theory was very important in theoretical modelling of light scattered by small particles. For particles with $a \ll \lambda$ Mie theory can be approximated by the Rayleigh scattering theory which is much simpler than the full Mie theory. The Rayleigh theory will not be discussed in length here as this work deals with particles too large to be approximated by Rayleigh theory. It will be noted though, that the normalized phase function $p(\theta)$ for Rayleigh scattering is quite simple

$$p_{Rayleigh}(\theta) = \frac{3}{16\pi}(1 + \cos^2 \theta).$$

Thus the angular distribution of the scattered irradiance is proportional to $1 + \cos^2 \theta$. It is symmetric in the forward and backward directions and varies only from 2 (at 0° and 180°) to 1 (at 90°). At 90° the $i_\parallel$ component vanishes and the scattered radiation is therefore plane polarized perpendicular to the scattering plane.

For particles with $a \gg \lambda$ the Mie theory can be approximated by the so called geometrical optics. The technique of geometrical ray-tracing (or geometrical optics) can be used
4.3. SCATTERING BY A SIZE DISTRIBUTION

in particular cases to obtain accurate numerical results for very large particles. However, probably the most useful function of such numerical computations is the physical explanation they provide for many of the features in light scattered by particles of size much larger than the wavelength of the incident light. Geometrical optics will not be discussed here in detail. See Hansen and Travis [1974] for a brief discussion or van de Hulst [1957] for a detailed description.

4.3 Scattering by a Size Distribution

In the previous sections we discussed scattering of light by identical particles. This section discuss scattering by a sample of non-identical particles, in the sense that all the particles are spheres and consist of the same material but have different sizes. This is a more realistic situation as natural particles usually are described by a size distribution. There are many ways of describing a size distribution; examples are gaussian and log-normal distributions. In Figure 4.9 gaussian and log-normal distributions are plotted as functions of the particle radius. To the left two gaussian distributions are plotted as functions of the particle radius around a mean value of \( \mu = 1.0 \ \mu m \). The standard deviation \( \sigma \) for the two curves are 0.05 \( \mu m \) and 0.1 \( \mu m \), respectively. Right, two log-normal distributions around a mode of 1.0 \( \mu m \). The log-normal standard deviation for the curves are 1.71 \( \mu m \) and 1.30 \( \mu m \) respectively. In this example the width of the log-normal distribution is much wider than the width of the gaussian distribution.

Figure 4.9: Two examples of particle size distributions. Left, gaussian (or normal) distributions around a mean value of \( \mu = 1.0 \ \mu m \). The standard deviation \( \sigma \) for the two curves are 0.05 \( \mu m \) and 0.1 \( \mu m \), respectively. Right, two log-normal distributions around a mode of 1.0 \( \mu m \). The log-normal standard deviation for the curves are 1.71 \( \mu m \) and 1.30 \( \mu m \) respectively. In this example the width of the log-normal distribution is much wider than the width of the gaussian distribution.

the particle radius around a mean value of \( \mu = 1.0 \ \mu m \), with a standard deviation \( \sigma \) of 0.05 \( \mu m \) and 0.1 \( \mu m \). These values have been chosen as they are the standard deviation found in samples of polyurethane microspheres used in a scattering experiment, discussed in chapter 5. In Figure 4.9 (right), log-normal distributions are shown with a standard deviation of 1.71 \( \mu m \) and 1.30 \( \mu m \) for the mode \( \mu = 1.0 \ \mu m \). The mode of the log-
normal distribution is defined as the particle radius where the curve has its peak (where the derivative is zero). This is not the mean value as can be seen by simple inspection of the curves. The standard deviation for the log-normal distribution $\sigma = 1.71 \ \mu m$ is found in many log-normal size distributions of dust found in ice cores from Greenland, see Steffensen [1997]. For comparison a distribution with a standard deviation of 1.30 $\mu m$ has also been plotted. One cannot directly compare the width of the two distributions (gaussian and log-normal) by comparing $\sigma$ as the standard deviation for the log-normal distribution is not defined for $\sigma < 1$. However, in this example it is evident from the scale of the x-axes that the log-normal distributions are much wider than the gaussian distributions.

The gaussian distribution has the form

$$N(a) = \exp \left( -\frac{1}{2} \left( \frac{a - \mu}{\sigma} \right)^2 \right),$$

where $a$ is the particle radius, $\mu$ the mean value, and $\sigma$ the standard deviation. The form of the log-normal distribution is

$$N(a) = \frac{1}{\sqrt{2\pi} \ln \sigma} \frac{1}{a} \exp \left( -\frac{1}{2} \left( \frac{\ln a - \ln \mu}{\ln \sigma} \right)^2 \right),$$

where $\mu$ is the log-normal mode, and $\sigma$ is the log-normal standard deviation. For both distribution we have

$$\int_{a_{\text{min}}}^{a_{\text{max}}} N(a) da = 1,$$

to ensure the scattered irradiance from the size distribution is consistent with the scattered irradiance from a single particle.

The effects on the extinction efficiency $Q_{\text{ext}}$ for a size distribution with different standard deviations can be seen in Figure 4.10. The extinction efficiency for a non-absorbing particle can be seen as the black curve in Figure 4.10. The same curve was plotted in Figure 4.4. As the incident light is scattered by a size distribution the distinct features of the single scattering pattern will be smoothed by the contribution of different sizes of the scatterers. The wider the size distribution is the fewer features the scattering pattern will show. The ripple structure seen for a single particle is the first to disappear as the standard deviation $\sigma$ increases. As the distribution is further widened the interference structure (large maxima and minima) fades away. The disappearance of the distinct features of the single scattering pattern is also evident in Figure 4.5 where the asymmetry factor is shown for a single particle and for a gaussian size distribution.

Figure 4.11 shows the effect on the scattering pattern of unpolarized light scattered by a gaussian dust distribution with an increasing standard deviation; both distributions have a mean particle size of 1.0 $\mu m$. The incident light has a wavelength of 632 nm and
the scattering pattern is that for spherical dust particles embedded in ice. Again the wide size distribution smoothes out the interference structure, although some undulations exist in the near backscattering region.

In section 3.2 above, the possible range in scattering angles for the Cryobot geometry were derived for all the permissible configurations of the Laser Dust Detector. The range in scattering angles were found to be \([\psi_1, \psi_2] = [78.1^\circ, 165.2^\circ]\). The task of optimizing the laser incidence angle for the Cryobot geometry consists of choosing a small range in scattering angles (the size of the range must be equal to the field of view of the camera = \(23.8^\circ\)) where the intensity of the scattered light is highest and most uniform. In Figure 4.11 the permissible range is shown as well as our choice for the best range, \(\psi_1 = 100.0^\circ\) and \(\psi_2 = 123.8^\circ\). Within the permissible range of scattering angles this is where the curve is most uniform. According to Figure 3.6 this corresponds to an incidence angle of the laser beam in the Cryobot hull, \(\theta_{\text{hull}} = 60.3^\circ\).

By modeling Mie scattering on a size distribution the scattering pattern is found which enables us to choose the best range in scattering angles and thereby setting the laser incidence angle for the optimal Cryobot configuration.
4.4 Non-spherical Particles

Because homogeneous, spherical particles are the exception in nature, rather than the rule, various exact and approximate methods have been developed for determining the absorption and scattering from non-spherical particles, including those with a smooth regular surface as well as those with more irregular shapes. Rayleigh theory can be applied to small non-spherical particles, and geometrical optics combined with diffraction theory can be applied to large non-spherical particles, see e.g. van de Hulst [1957] and Bohren and Huffman [1983]. When the size of the particles compares to the wavelength, difficulties arise and it is in this size limit that most recent theoretical efforts on light scattering on non-spherical particles have been directed. Although it might be possible to derive the scattering and absorption by numerical methods, computational time can be excessive and efficient methods for computing scattering by non-spherical particles are actively being developed. Yeh and Mei [1980] have given an overview of some of the earlier methods. A few of the more recent developments are discussed below.

Scattering properties of particles with sizes comparable to the wavelength (also called resonance particles) are complicated functions of the particle size parameter, shape, and
refractive index. Perhaps the fastest and most powerful numerical method for rigorously computing non-spherical light scattering in the resonance region of size parameters is the T-matrix method [Waterman, 1971]. Mishchenko et al. [1997] used the T-matrix method to extensively compute light scattering by shape distributions of polydisperse, randomly orientating spheroids with refractive indices and size distributions representative of naturally occurring dust aerosols. They found that even after averaging over size and orientation, a single spheroidal shape always produced a unique phase function, specific for that shape, and distinctly different from other spheroidal shapes. However, by averaging over a wide distribution of prolate and oblate spheroids, they found that the phase functions were smooth and featureless and closely resembled those measured for natural soil and dust particles. This showed that, although natural dust particles are not perfect spheroids, optically their mixture of highly variable shapes results in a phase function that could be adequately modeled using a wide aspect-ratio distribution of prolate and oblate spheroidal particles.

Mishchenko et al. [1997] found that differences in the scattering phase function of non-spherical (T-matrix method) and spherical particles (Mie method) could be very large, but the differences in the asymmetry factor of the phase function and the fraction of backscattered light were in most cases smaller than 10%.

Volten et al. [2001] measured scattering matrices as functions of the scattering angle for seven distinct irregularly shaped aerosol samples (feldspar, red clay, quartz, loess, Pinatubo and Lokon volcanic ash, and Sahara sand) with properties representative of mineral aerosols present in the Earth’s atmosphere. They found, consistent with Mishchenko et al. [1997], that the measured scattering matrix, plotted as functions of the scattering angle, were rather similar and that all the measured phase functions were smooth with virtually no structure for side scattering and back scattering angles (between roughly 50° and 170°). The shapes were similar for all aerosol samples. This similarity in scattering behavior led the authors to construct an average aerosol scattering matrix as a function of the scattering angle for a general atmospheric aerosol with average optical parameters.

The theoretical modeling performed in this work have all been computed via Mie scattering on spherical particles. Light scattering on non-spherical particles is an entire work in itself and the possible improvement in accuracy in the numerical analysis on light scattering would not be justified by the time spent implementing the necessary changes in the computer codes and the time consuming numerical computations, especially taking into account the uncertainty in particle shapes in the ice sheets even on Earth [Donarummo, 1997].
CHAPTER 4. SCATTERING OF LIGHT BY SMALL PARTICLES
Chapter 5

Laboratory Experiments on Mie Scattering

5.1 The Single Scattering Domain

All scattering experiments in this work, including simulated Mie scattering and experimental work in the laboratory, are assumed to take place in the single scattering domain. Strictly speaking, in this domain a single photon is emitted by the laser, scattered by one particle (and one particle only), and then detected by an appropriate detector. This is a very strict constraint and in reality the condition of a single scattering domain is fulfilled if the fraction of single scattered photons is much much larger than the fraction of multiple scattered photons. Imagine a collection of particles in a liquid suspension irradiated by light from a laser, and imagine that the width of the laser beam is large enough to irradiate all the particles under consideration. Generally speaking the particles in the suspension are electromagnetically coupled: each particle is excited by the external laser light plus the resultant field scattered by all the other particles. In this situation the field scattered by the particle depends on the total field to which the particle is exposed, i.e. the applied field from the laser plus the field arising from the interaction of the neighboring particles with the external field. By assuming single scattering, however, considerable simplification is achieved. In this case the number of particles is sufficiently small and their separation sufficiently large that, in the neighborhood of any particle, the total field scattered by all the particles is small compared with the external field from the laser. It is difficult to state precise general conditions under which the single scattering criterion is satisfied. The assumption basically requires that the interference of light scattered by different particles is undetectable (i.e. at the noise level). van de Hulst [1957] states as a rough rule of thumb that the particles must be separated by a few times their radius and also points out that "A simple and conclusive test for the absence of multiple
scattering is to double the concentration of particles in the investigated sample. If the scattered intensity is doubled, only single scattering is important.”. This is the basis for the following experiment where we seek the upper limit on the number of scatterers per volume allowed in the single scattering domain, for our specific geometry, particle size and light wavelength.

5.1.1 Experimental Setup

Laser light is directed into a beaker filled with a suspension of microparticles at a precisely known concentration. The scattered signal is then measured for an increasing number of microparticles per volume (in the following called the concentration of microparticles). Next the scattered signal is plotted as a function of the concentration. As van de Hulst [1957] points out, a linear relationship between the scattered signal and the concentration must exist in the single scattering regime. The following experiment seeks to find the number of particles per milliliter where this linear relationship breaks down and we cross into the multiple scattering domain.

A sketch of the setup is shown in Figure 5.1. The laser is mounted on an arm fixed on a precision rotation table. A beaker is positioned at the center of rotation for the arm and laser. The camera has a fixed position and the optical axis crosses the rotation axis of the arm and laser. This way the camera always sees the same point in the center of the beaker. A circular mask reducing the field of view of the camera is placed in front of the lens (not shown in Figure 5.1). This mask has been designed as a cone-shape to make sure that light hitting the mask will not be reflected back into the beaker. In the beaker small latex spheres are suspended in ultra pure distilled water\(^1\). The latex spheres are usually used as calibration for particle counters and are therefore calibrated with a known narrow size distribution (see Appendix A for a more detailed description of the microspheres).

The microparticles used in the experiment have a mean radius of 0.75 µm with a standard deviation of 0.02 µm. They are delivered from the manufacturer in a 2.6% aqueous solution contained in a small dropper bottle. The suspension of microparticles is prepared in a 1500 ml container filled with ultra pure water. To this container is added one drop of particles in the supplied 2.6% aqueous solution. The volume of one drop has previously been determined by measuring the volume of 3 sets of 100 drops and then averaging the 3 sets. The result can be seen in Table 5.1, i.e. an average of 4.15 ml per 100 drops of microparticles giving \(4.15 \times 10^{-2}\) ml/drop.\(^{\text{18 MΩ filtered, distilled and deionized water}}\) After the particles have been evenly distributed in the 1500 ml pure water by stirring, a small portion of the suspension is transferred to the beaker in the setup. Images are taken at the 3 different scattering

\(^1\)8 MΩ filtered, distilled and deionized water
5.1. **THE SINGLE SCATTERING DOMAIN**

![Experimental Setup](image)

**Figure 5.1:** The experimental setup used in determining the upper limit for the concentration of scatterers allowed in the single scattering domain. This experiment is performed at 3 different scattering angles ($45^\circ$, $90^\circ$, and $135^\circ$). For illustration purposes a setup with a scattering angle of $160^\circ$ is showed.

<table>
<thead>
<tr>
<th>Set</th>
<th>Volume/100 drops</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.15 ml</td>
</tr>
<tr>
<td>2</td>
<td>4.20 ml</td>
</tr>
<tr>
<td>3</td>
<td>4.10 ml</td>
</tr>
<tr>
<td>Average</td>
<td>4.15 ml</td>
</tr>
</tbody>
</table>

**Table 5.1:** Determination of the volume of 100 drops of microparticles.

The laser shines into the beaker and for three different scattering angles $45^\circ$, $90^\circ$, and $135^\circ$ (between the incident laser and the optical axis) an image is taken of the scattered light. The first 3 images (one for each angle) are taken with no particles in the water to get a reference signal for the water only. The water has been filtered with a 0.2 μm filter.
and any scattering in the water will come from particles smaller than this. This putative scattering will be dominated by Rayleigh scattering that is much more isotropic than Mie scattering from larger particles. The isotropic nature of the Rayleigh scattering will only provide a relatively flat background to the highly non-isotropic Mie scattering from larger particles.

In Figure 5.2 two negative data images are shown. The laser enters the field of view of the camera (mounted with the circular mask in front of the lens) at the left side and exits at the right side of the image. The image shows the laser beam being scattered by particles in the beaker. In the left image the concentration is low enough to ensure single scattering. Only photons that are scattered once and detected by the camera are seen. In the right image however, the multiple scattering domain is reached which is evident by the scattering of light outside the physical boundary of the laser beam. The photons that cause darkening outside the boundary of the beam have been scattered more than once and are therefore characteristic of multiple scattering.

Figure 5.3 shows 4 normalized light intensity profiles of 4 images similar to the images in Figure 5.2. These curves profile an image from top to bottom and have been made by averaging 50 columns in the middle of the image. The figure shows the normalized light intensity profile of four images with increasing concentration of microparticles. In theory the vertical profile of an image in the single scattering domain would be a box function that is zero outside the physical boundary of the laser beam and non-zero inside the boundary. In reality the curve has a small shoulder even for the smallest concentrations because a profile of the laser beam itself is not a box function. It more resembles a narrow gaussian. The black curve in Figure 5.3 (at $4 \times 10^5$ particles/ml) is for a low concentration of microparticles. As the concentration increases so does the shoulder and for concentrations characteristic of multiple scattering (the blue and red curves at 70 and $120 \times 10^5$ particles/ml, respectively) the curve is non-zero everywhere inside the field of
5.1. **THE SINGLE SCATTERING DOMAIN**

![Normalized 'vertical' image profiles at four different concentrations of microparticles.](image1)

Figure 5.3: Normalized 'vertical' image profiles at four different concentrations of microparticles.

view of the camera.

Figure 5.4 (left) shows the total scattered signal received signal as a function of the concentration of microspheres for two different angles 90° and 135°. Note the linear relationship for small concentrations (below $35 \times 10^5$ particles/ml). In this linear region single scattering is dominant. As mentioned in the preceding section single scattering

![The total scattered signal received as a function of the concentration of microspheres for two different angles 90° and 135°.](image2)

Figure 5.4: Left: The scattered signal as a function of the concentration of microparticles for two scattering angles (90° and 135°). The vertical dotted line marks a reasonable value for the concentration where the linear relationship begins to break down. Right: The scattered signal vs. concentration for a 45° scattering angle. The lines represent linear fits to the first 9, 16, and 24 points, respectively.
is, in reality, characterized merely by a much larger fraction of single scattering photons than multiple scattering photons. As the scattering is increased, with increasing particle concentration, the fraction of multiple scattered photons will increase monotonically until this fraction dominates and the multiple scattering domain is reached. Exactly where the line between single and multiple scattering is crossed therefore depends on the accuracy required of the measurement, i.e. it is not a fundamental parameter. In Figure 5.4 the single scattering assumption breaks down when the linear relationship breaks down, and by visual inspection this occurs at about $35 \times 10^5$ particles/ml, marked by the vertical dotted line in the left figure and the number 9 in the right figure. Figure 5.4 (right) shows the scattered signal vs. concentration for scattering at $45^\circ$. This is the same curve as in Figure 5.4 (left) but for a scattering angle of $45^\circ$.

The method used to determine more precisely where the single/multiple scattering line is crossed is by fitting a straight line to an increasing number of data points and plotting the $\chi^2$ - goodness of fit as a function of particle concentration. In Figure 5.4 (right) examples of three such fits can be seen. The black line fits to the first 9 data point, the red and green lines fit to the first 16 and 24 data points, respectively. By calculating $\chi^2$ for a linear fit to an increasing number of data points and then plotting $\chi^2$ as a function of the concentration, one gets a better idea of where the single scattering assumption breaks down. Starting with a linear fit to the first two points (which obviously give a $\chi^2$ of zero) and ending with a linear fit to all the data points (which gives a very poor $\chi^2$) enables us to estimate the concentration where we no longer can assume single scattering. This is shown in Figure 5.5 for the three examined scattering angles and for their average (the black curve with squares). The vertical arrow represents a reasonable (although strict) choice for the value of the concentration where the single scattering domain ends and the multiple scattering domain begins. This happens at a concentration $c_{\text{limit}}$ of

$$c_{\text{limit}} = 35 \times 10^5 \text{ particles/ml}.$$ 

As discussed earlier $c_{\text{limit}}$ is not a firm value but depends on the accuracy required of the measurements. $c_{\text{limit}}$ corresponds to the green curve in Figure 5.3 and it is evident from the non-zero shoulder of the green curve that a small fraction of multiple scattering exists. The fraction is much smaller than in the multiple scattering domain, though.

**Particle Concentration in Greenland Ice Cores**

Steffensen [1997] examined the size distribution of microparticles (dust) from selected segments of the Greenland Ice Core Project (GRIP) ice core. He found dust masses ranging from a few tens of $\mu$g of dust per kg of ice to an upper limit of 7600 $\mu$g/kg, and found that these particles had a log-normal distribution in size. In Appendix B these dust
5.1. **THE SINGLE SCATTERING DOMAIN**

Figure 5.5: The $\chi^2$ - goodness of fit plotted as a function of the concentration of microparticles for the three different scattering angles and for their average (black curve with squares).

Masses are converted to number of particles per milliliter where due account is taken of the mean radius of the particles in the given depth interval.

Figure 5.6 shows the number of particles per milliliter at the sampled depth\(^2\). In the following, experiments done on several different Greenland ice cores will be discussed. In the

\[^2\text{Data taken from Steffensen [1997], Table 1}\]
5.2 Experimental Verification of the Mie Model

In this section an experiment is described that seeks to verify the Mie method, discussed previously. Obviously it is the experimental setup used in this work that is verified, as the Mie model itself has proved its validity extensively in the past nearly 100 years.

5.2.1 Experimental Setup

The same experimental setup used in determining the limit for single scattering is used in this experiment (see Figure 5.1). For this experiment the concentration is fixed at a constant value, in the single scattering domain, and the scattering angle is changed, from 20° to 158° in 2° steps. As the scattering angle is increased, or decreased, from 90° the path length of the laser through the beaker, as seen in the field of view of the camera, increases, see Figure 5.7. Therefore more light is scattered into the camera as a longer path length encounters more particles. To correct for this error a sin θ factor must be multiplied to the returned data, where θ is the scattering angle.

Specifics of the experimental setup can be seen in Table 5.2. In Figure 5.8 the scattered signal is plotted as a function of the scattering angle. The theoretical curve for the Mie scattering is plotted for comparison (red curve). Errorbars (∓sqrt(signal)) have been added to each data point. For scattering angles smaller than 20° the incident non-scattered laser beam will enter the field of view of the camera directly and drown out the scattered signal;

Table 5.2: Specifications of the parameters for the setup. The laser is vertically polarized 100:1 and the vertical dimension of the beam is 2 mm.

<table>
<thead>
<tr>
<th>Particle:</th>
<th>mean radius</th>
<th>standard deviation</th>
<th>concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05 µm</td>
<td>0.048 µm</td>
<td>29 × 10^5 particles/ml</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Laser:</th>
<th>wavelength</th>
<th>polarization</th>
<th>beam size</th>
</tr>
</thead>
<tbody>
<tr>
<td>632 nm</td>
<td>100:1</td>
<td>2 × 0.5 mm</td>
<td></td>
</tr>
</tbody>
</table>

these experiments an assumption of single scattering is made. This assumption is now found to be valid as the concentration of microparticles in the GRIP ice cores has a maximum of 6.5 × 10^5 particles/ml. This is only about 20% of the upper limit for single scattering. The Antarctic ice sheet also constitutes a single scattering environment as it is generally less dusty than Greenland ice [Petit et al., 1999] while the dust content in the ice of Mars and Europa is not known.
5.2. EXPERIMENTAL VERIFICATION OF THE MIE MODEL

As the path length of the laser beam increases for angles different from 90°, \( \sin \theta \) must be multiplied to the returned data, where \( \theta \) is the scattering angle.

The scattered signal for angles smaller than 20° will therefore not be plotted. For angles larger than 158° (and less than 180°) the laser beam will hit the outer end of the camera lens and be stopped before entering the beaker, see Figure 5.1. If the camera were moved further away from the beaker, it would be possible to detect scattered light from angles larger than 158°, but the resolution of the images would decrease. A compromise has therefore been set at the given distance and maximum scattering angle which is within the range of angles used in the Laser Dust Detector developed in this work. The scattered signal compares very well to the Mie theoretical model and for this size distribution with a very narrow width (\( \sigma = 0.05 \) \( \mu m \)) the experimental method is hereby verified.

As the experimental method needs to be verified for analyzing dust embedded in ice cores the following experiment compares the scattered signal from a rather wide size distribution to that of the corresponding theoretical Mie model. The size distribution used in this experiment is produced by adding three drops of microparticles to a container filled with 600 ml ultra pure water. The three drops have mean sizes 0.77 \( \mu m \), 1.04 \( \mu m \), and 1.59 \( \mu m \). A plot of the normalized size distribution for these particles can be seen in Figure 5.9. The standard deviation for the three sizes (0.77 \( \mu m \), 1.04 \( \mu m \), and 1.59 \( \mu m \)) used in producing the size distribution is 0.020 \( \mu m \), 0.048 \( \mu m \), and 0.078, respectively. The
Figure 5.8: The scattered signal plotted as a function of the scattering angle. For comparison the Mie model is plotted for the same parameters as described in Table 5.2. The errorbars for the experimental data is $\pm \sqrt{\text{signal}}$.

Figure 5.9: The size distribution used for verification of the experimental method for a polydisperse sample of microparticles. The size distribution has been normalized to be able to compare the scattered signal to that from a single particle.

scattered signal plotted as a function of the scattering angle can be seen in Figure 5.10. Again the scattered signal compares very well to the Mie model. The value returned from the Mie model is irradiance [W/m$^2$] and the value returned from the image is integrated photons scaled to 16 bit (0-65535). In both the above experiments the scattered signal has therefore been scaled to the simulated Mie data at 90° scattering angle. In section 5.4
5.2. EXPERIMENTAL VERIFICATION OF THE MIE MODEL

Figure 5.10: The scattered signal from a size distribution compared to the corresponding Mie model. The errorbars for the experimental data is $\pm \sqrt{\text{signal}}$.

below, a calibration of the two units is performed. With these scattering experiments completed and the results compared to the theoretical Mie model, we are confident in our experimental method.

5.2.2 Suspension Times for the Microspheres

To get an accurate value of the scattering of the laser light by the microspheres in the beaker it is important that the particle distribution is homogenous, and that it stays homogenous throughout the duration of the experiment (maximum 2 hours). Below follows a discussion of the suspension time for the microspheres.

By using Stoke’s law the settling velocity of the spheres can be found and from that the suspension time is given. Stoke’s law relates the settling velocity of a smooth, rigid sphere in a viscous fluid of known density and viscosity to the diameter of the sphere when subjected to a known force field:

\[
V = \frac{2 g a^2 (\rho_p - \rho_w)}{9 \eta},
\]

$V =$ settling velocity $[\text{m s}^{-1}]$,
$g =$ gravitational acceleration $= 9.81 \text{ m s}^{-2}$,
$a =$ radius of particle $= 1.05 \times 10^{-6} \text{ m}$,
$\rho_p =$ density of particle $= 1050 \text{ kg m}^{-3}$,
$\rho_m =$ density of water $= 1000 \text{ kg m}^{-3}$,
$\eta =$ viscosity of water $= 1.0 \times 10^{-3} \text{ N s m}^{-2}$. 

Thus the settling velocity is

\[ V = 1.2 \times 10^{-7} \text{ m s}^{-1} = 0.43 \text{ mm/hour}. \]

This means that during the 2 hours (maximum) that the experiment lasts, a particle will fall less than half the vertical height (2 mm) of the laser beam. This is acceptable and poses no problem to the homogeneity of the suspension of particles.

### 5.3 Radiometry for the Laser Dust Detector

In the following an estimate is made of the number of scattered photons needed for a given optical system to have a 'good' signal to noise ratio. Thereafter, a calculation for our proposed system is made and it’s found that there are an abundant number of photons reaching the sensitive area of the CCD.

#### The CCD Detector

Silicon in crystalline form is made up of silicon atoms, covalently bonded to their neighbors. The electrons in the silicon are bound to the structure and are therefore not readily detectable. However, if energy is supplied to an electron that is greater than the band gap energy of silicon (about 1.1 eV) an electron-hole pair is created. The electron is then promoted into the conduction band and can easily be detected. The energy needed to create this electron-hole pair can be supplied by photons that have a wavelength shorter than \( \sim 1 \mu\text{m} \) (if the energy needed is \( E = 1.1 \text{ eV} = 1.8 \times 10^{-19} \text{ J} \), then the wavelength must be shorter than \( \lambda = \frac{hc}{E} = 1.1 \mu\text{m} \)). Thermal energy can also have sufficient energy to create free electrons and these are indistinguishable from photoelectrons. The electrons created are collected and stored in regions on the silicon chip that are defined by an electric field. These regions are called pixels. The electric field defines a potential well under which the free electrons can be collected. The potential well capacity of electrons ranges from tens of thousands up to a million electrons. The full well capacity (FWC) of the CCD\(^3\) used in our setup is more than 70,000 electrons.

The CCD detector used in this work is in a digital astronomical camera with internal single stage Peltier cooling. The CCD chip reaches a temperature about 30°C below ambient temperature and has a very low noise level (or 'dark current') due to thermally exited free electrons. According to the manufacturer the readout noise is on average (RMS) 8 electrons and the dark current at room temperature is 0.1 electrons/pixel/s.

The quantum efficiency (QE) is the probability of an incident photon generating a photoelectron in the CCD sensor. It is normally expressed an a percentage, hence 10%.

---

\(^3\)The CCD chip in the camera is a SONY EX View ICX249AL
5.3. RADIOMETRY FOR THE LASER DUST DETECTOR

QE will on average only generate one photoelectron in ten incident photons, and 100% QE is the total certainty that an incident photon will generate a photoelectron in the CCD sensor. Figure 5.11 shows the quantum efficiency vs. wavelength for our CCD spanning the range from UV to near IR\(^4\). As can be seen the QE is reasonably high and peaks at about 64% at the wavelength of the laser beam used (632 nm) in the experimental setups.

![Figure 5.11: The quantum efficiency of the CCD chip in the camera used throughout this work. The horizontal dotted line represents the QE (∼ 64%) at the wavelength of the laser beam (632 nm).](image)

**How Much Light Do We Need?**

Assuming a maximum noise level of the CCD chip of about 10 electrons at room temperature and assuming a quantum efficiency of 64% we’ll only need 16 photons per pixel to reach the noise level. Assuming a signal to noise ratio of 10-100 we’ll need 10 to 100 times as many photons which amount to 160-1600 photons needed per pixel. The CCD chip has 752×580 pixels thus totalling \(7 \times 10^7 - 7 \times 10^8\) photons needed at the sensitive area of the CCD. This number will be compared with the actual number reaching the CCD for the given optical design.

**How Much Light do we Have?**

The laser beam intended for the field deployed geometry will have a nominal power of 25 mW. Naturally, lasers with more power can be used but 25 mW is a good trade-off between having enough power on one hand and cost and safety on the other. The light

\(^4\text{Data supplied by the manufacturer.}\)
arriving at the CCD is the scattered irradiance (in Wm\(^{-2}\)). The laser is polarized 100:1 and can be focussed. Defocussing the laser will not produce a circular beam cross section but will expand the beam in one plane more than in the perpendicular plane, according to the 100:1 polarization. This makes the cross section of the beam rectangular and the preferred beam size is 1.5 mm x 0.5 mm. Maximum focussing of the beam will make the width of the beam too small, as a certain statistical averaging of the scattering from numerous particles is wanted. These particles will not be fixed in the suspension but will slowly settle over time. A very narrow beam will have a tendency to scintillate as the suspended particles move in and out of the beam. The preferred beam size gives a cross sectional area of the beam of 1.5 mm \(\times\) 0.5 mm = 7.5 \(\times\) 10\(^{-7}\) m\(^2\), assuming the divergence of the beam is negligible. The irradiance \(I\) of the incident laser beam is therefore

\[
I = 25 \times 10^{-3} \text{W}/7.5 \times 10^{-7} \text{m}^2 = 3.3 \times 10^4 \text{W/m}^2.
\]

A given particle with a scattering cross section of \(C_{\text{sca}}\) (in m\(^2\)) completely embedded in the laser beam will scatter an electromagnetic power \(P\) (in W) of \(P = 3.3 \times 10^4 \text{W/m}^2 \cdot C_{\text{sca}}\) in all directions. If the particle is absorbing however (having a complex refractive index), we have \(C_{\text{ext}} = C_{\text{abs}} + C_{\text{sca}}\), thus the particle will remove the power

\[
P = I \cdot C_{\text{ext}}
\]

from the laser beam by extinction, i.e. by scattering and absorption. The particle will scatter the incident light in all directions, with the detailed scattering pattern as discussed in section 4.2 above, but the CCD will not detect all the scattered light in 4\(\pi\); it will only detect the light scattered into the sensitive area of the detector. The phase function \(p(\theta)\) (in sr\(^{-1}\)) is the probability for scattering in the direction of \(\theta\). Therefore, giving the aperture area of the detector \(A\) (in m\(^2\)) and the distance to the detector \(r\) (in m), the electromagnetic power scattered in the direction of the detector (here called \(\theta\)), for the solid angle \(A/r^2\), is

\[
P = I \cdot C_{\text{ext}} \cdot p(\theta) \cdot \frac{A}{r^2}.
\]

(5.1)

Mie computer calculations return the parameters \(S_1, S_2\) (in (4.13) and (4.14)) as well as \(Q_{\text{ext}}\), and we have \(C_{\text{ext}} = GQ_{\text{ext}}\), where \(G = \pi a^2\) is the cross sectional area of the particle and \(Q_{\text{ext}}\) the extinction efficiency. The phase function \(p(\theta)\) is given by Bohren and Huffman [1983], Eq. 13.3, p. 384

\[
p(\theta) = \frac{S_{11}(\theta)}{k^2 C_{\text{ext}}},
\]

where

\[
S_{11} = \frac{1}{2}(|S_1|^2 + |S_2|^2)
\]

according to (4.19) and \(k = 2\pi/\lambda\) is the wavenumber in the medium.
The Sensing Volume

The above derivations were performed for one particle. Assuming single scattering, (5.1) must be multiplied by the number of particles in the sensing volume to get the total power reaching the sensitive area of the CCD. The sensing volume is defined as the volume of the laser beam inside the field of view of the detector. That is, we want to find the number of particles \( N_p \) contributing to scattering light into the sensitive area of the detector. Referring to Figure 5.12 the sensing volume can be divided into two parts separated by the line \( f \). The length of the left part is determined by using the sine relation on line \( d \) and the angles \( \gamma \) and \( \epsilon \). The length of the right part of the sensing volume is determined by using the sine relation on the line \( d' \) and the angles \( \delta \) and \( \beta \). With the following parameters \( f = 50 \text{ mm}, \alpha = 45^\circ \), and \( \phi = 31.3^\circ \) the angles are calculated as follows

\[
\gamma = 180^\circ - \phi/2 - (\alpha + 90^\circ) = 29.4^\circ \\
\epsilon = 180^\circ - \alpha - \gamma = 105.6^\circ 
\]

Figure 5.12: A sketch of the geometrical setup showing the sensing volume. Equation (5.1) will be multiplied by the number of particles inside the sensing volume.
\[ \beta = 180^\circ - \phi/2 - 90^\circ = 74.4^\circ \]
\[ \delta = 180^\circ - \alpha - \beta = 60.6^\circ. \]

The length of line \( d \) is
\[ d = d' = f \cdot \tan(\phi/2) = 14.0 \text{ mm}, \]
and the length \( s \) of the sensing volume is then determined as the addition of the two sine relations of the left and right part:
\[ s = \frac{d \sin \epsilon}{\sin \gamma} + \frac{d' \sin \beta}{\sin \delta} = 42.9 \text{ mm}. \]

The sensing volume \( V_s \) is the length \( s \) times its cross sectional area, thus
\[ V_s = s \cdot 1.5 \text{ mm} \cdot 0.5 \text{ mm} = 32 \text{ mm}^3. \]

**The Total Power Reaching the CCD**

For Greenland ice sheet ice the mass of dust particles in a retrieved ice core from GRIP, Greenland ranges from 100 \( \mu \text{g} \) to 7600 \( \mu \text{g} \) of dust in 1 kg of ice [Steffensen, 1997]. The number density of dust particles in GRIP ice is found in Appendix B and for Greenland ice sheet dust mass the number of particles in 1 mm\(^3\) of ice ranges from 9 to 670 particles. Therefore, with a sensing volume of 32 mm\(^3\) the range in the total number of particles in the sensing volume is 288 to 21,440 particles.

Assuming that these particles are homogeneously distributed in the sensing volume, every particle will have a unique scattering angle \( \theta_i \) as symbolized by the red arrows pointing toward the detector in Figure 5.13. The scattering angle \( \theta_i \) is, from the sine relation, given as
\[ \theta_i = \arcsin\left(\frac{b_i \sin \gamma}{r_i}\right), \]
where \( r_i \) is the distance of the \( i \)th particle to the detector. For the values of \( f, \phi, \) and \( \alpha \) noted above, the range in scattering angle, within the field of view of the camera, is
\[ \theta \in [119.4^\circ ; 150.7^\circ]. \]

Therefore, to estimate the total power reaching the detector we sum the phase function \( p(\theta_i) \) over the range in scattering angles \( \theta_i \), the extinction cross section \( C_{\text{ext}}(a_i) \) over the particle radius \( a \) and the solid angle \( A/r_i^2 \) over the distance \( r_i \) from the \( i \)th scatterer to the camera. The microspheres used in the experimental determination of the limit for single scattering, described above, have a gaussian size distribution. It is here assumed that the collection of particles in the sensing volume will also have a gaussian size distribution. We want to find the total power reaching the sensitive area of the detector. It is given as
\[ P = I \cdot N_p \cdot C_{\text{ext}}(a) \cdot p(a, \theta) \cdot \frac{A}{r^2}, \]
5.3. RADIOMETRY FOR THE LASER DUST DETECTOR

where the extinction cross section explicitly depends on the particle radius $a$ and the phase function explicitly depends on the scattering angle $\theta$ and the particle radius. $N_p$ is the total number of particles in the sensing volume, and $A/r^2$ is the solid angle extended by the camera aperture.

In Figure 5.14 the phase function for a gaussian size distribution is plotted as a function of the scattering angle. The mean particle radius is 1.59 $\mu$m with a standard deviation of $\sigma = 0.156$ $\mu$m. Assuming that the particles are homogenously distributed in the sensing volume every particle contributes to the total scattered irradiance with a slightly different power because every particle has a different scattering angle and differs in size from most of the other particles (thus having a different $C_{ext}(a_i)$ and $p(a_i, \theta_i)$) and furthermore has a different distance to the detector (thus scattering into a different solid angle $A/r_i^2$).

As mentioned above, the number of particles range between 288 and 21,440 particles occupying the sensing volume. Therefore we sum over the number of particles and hence

Figure 5.13: A sketch of a homogenously distribution of particles in the sensing volume. Every particle will have a different scattering angle $\theta_i$ symbolized by a red arrow pointing toward the detector. Moreover it is assumed that the collection of particles will have a gaussian size distribution.
Figure 5.14: The phase function of a normalized gaussian size distribution of microspheres. The dotted vertical lines represent the range in scattering angles $\theta$ for the given experiment.

<table>
<thead>
<tr>
<th># of particles ($N_p$)</th>
<th>total power ($P$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>288</td>
<td>$1.6 \times 10^{-9}$ W</td>
</tr>
<tr>
<td>21,440</td>
<td>$1.2 \times 10^{-7}$ W</td>
</tr>
</tbody>
</table>

Table 5.3: The total power reaching the sensitive area of the detector. Particle numbers refer to the range of embedded particles in ice sheet ice from GRIP, Greenland [Steffensen, 1997].

The Integration Time

In this section we want to find the integration time of the camera in order to acquire an image with a 'good' signal to noise ratio. The basis for the calculations is the total power

$$P = I \cdot \sum_{i=1}^{N_p} C_{ext}(a_i) \cdot p(a_i, \theta_i) \cdot \frac{A}{r_i^2}.$$ 

Numerical computations are done for 288 and 21,440 particles, homogeneously distributed in the sensing volume, representing the whole range in scattering angles from 119.4° to 150.7°. For every particle, representing a unique scattering angle, a random size is applied to the particle, obeying the overall gaussian distribution in size. The calculated total power is shown in Table 5.3.
5.4. CALIBRATION OF THE CCD RESPONSE

$P$ reaching the camera for the range in particle numbers given in Table 5.3. The energy of one photon is $E = \frac{hc}{\lambda}$, where $h$ and $c$ are Planck’s constant and the speed of light in vacuum, respectively. The number of photons $N_{ph/s}$ with a wavelength of $\lambda = 632$ nm emitted per second in a laser beam of power $P$ (given in Table 5.3) is given by

$$N_{ph/s} = \frac{P \cdot \lambda}{h \cdot c} = \begin{cases} 5.1 \times 10^9 \text{ photons/s for } N_p = 288 \\ 3.8 \times 10^{11} \text{ photons/s for } N_p = 21,440 \end{cases}.$$

The number of photoelectrons $N_{e^-/s}$ generated in the CCD per second is given as the number of photons $N_{ph/s}$ per second times the quantum efficiency of the CCD at a wavelength of 632 nm. For our CCD the QE is 64%, thus the number of generated photoelectrons per second are $N_{e^-/s} = 0.64 \cdot N_{ph/s}$.

The pixels in the CCD array have to ’share’ these photoelectrons, thus every pixel gets

$$N_{e^-/s/pix} = \frac{N_{e^-/s}}{752 \cdot 580 \text{ pixels}} = \begin{cases} 7.5 \times 10^3 \text{ photoelectrons/s/pixel for } N_p = 288 \\ 5.6 \times 10^5 \text{ photoelectrons/s/pixel for } N_p = 21,440 \end{cases}.$$

The total number of pixels sharing the photoelectrons is in reality much smaller than 752x580 as the lens system focuses the photons in a smaller area on the CCD. The actual number of pixels sharing the photoelectrons vary with particle density, and laser beam size, thus making 752x580 an upper limit of the number of participating pixels. This makes the integration time, derived below, shorter and enables the camera to detect light from even fewer scatterers in the ice.

As mentioned previously we need 1600 electrons per pixel to get a signal to noise ratio of 100. With the above ‘production’ of $N_{e^-/s/pix}$ photoelectrons per second per pixel, the required integration time to reach 1600 electrons is

$$t = \frac{1600 \text{ electrons}}{N_{e^-/s/pix}} = \begin{cases} 0.2 \text{ s for } N_p = 288 \\ 2.9 \text{ ms for } N_p = 21,440 \end{cases}.$$

In this case the thermal electrons are negligible. These exposure times are easily manageable showing that there are ample photons reaching the sensitive area of the detector. The exposure times can be decreased even more by either increasing the power of the laser, increasing the opening of the diaphragm of the lens (it’s set at 10 mm diameter for the experiment), or allowing a signal to noise ratio of less than 100 (which was chosen rather arbitrarily).

5.4 Calibration of the CCD Response

The previous section utilized theoretical Mie scattering in calculating the number of photoelectrons generated in the CCD chip as a function of a number of parameters. The
parameters include the incident irradiance of the laser beam and the number of scatterers participating in the scattering of light. This section verifies the use of the model and calibrates the response of the CCD chip for scattering of laser-light on latex microspheres.

In section 5.1 the upper limit in concentration of scatterers in the single scattering domain was determined. A series of images were acquired of an increasing concentration of latex microspheres and the scattered signal reaching the camera was plotted as a function of the particle concentration (Figure 5.4). In this section a similar setup is used with the same preparation of a suspension of microspheres.

A beaker containing a suspension of microspheres is positioned at a distance of 68 mm from the camera with the center of the beaker in the optical axis of the camera. A circular mask is placed in front of the camera lens to minimize collection of light not scattered by the particles. This decreases the field of view of the camera to 13.4°. The incident laser beam is perpendicular to the optical axis of the imaging system and the range of scattering angles is \( \theta \in [83.3° ; 96.7°] \), see Figure 5.15. As the deviation of

![Diagram of laser geometry](image)

Figure 5.15: A sketch of a perpendicular scattering setup giving a range in scattering angles of \( \theta \in [83.3° ; 96.7°] \). The distance \( r \) is assumed to be constant.
5.4. CALIBRATION OF THE CCD RESPONSE

the scattering angle from 90° is small the distance \( r \) from the particles to the detector is assumed constant \( (r = r_i = 68 \text{ mm}) \). With an angular field of view of 13.4° and a distance of 68 mm the length of the sensing volume is found to be 16 mm. With a laser beam cross section of 1.5 mm x 0.5 mm the sensing volume \( V_s \) is

\[
V_s = 16 \text{ mm} \cdot 1.5 \text{ mm} \cdot 0.5 \text{ mm} = 12 \text{ mm}^3.
\]

The particles used have a gaussian size distribution with a mean radius of \( a = 1.59 \mu\text{m} \) and a standard deviation of \( \sigma = 0.156 \mu\text{m} \). In Appendix A the number concentration of a suspension of microspheres with a mean radius of \( a = 0.75 \mu\text{m} \) is determined. A similar derivation of the larger particles of \( a = 1.59 \mu\text{m} \) gives a concentration \( c \) of

\[
c = n_d \cdot 4.0 \times 10^4 \text{ particles per ml},
\]

where \( n_d \) is the number of drops of latex particles (supplied in a 2.6% aqueous solution) added to a 1500 ml container, see section 5.1 for further details. Thus, the number of scatterers in the sensing volume is given as

\[
N_p = n_d \cdot 4.0 \times 10^4 \text{ particles/ml} \cdot 10^{-3} \text{ml/mm}^3 \cdot 12 \text{ mm}^3 = n_d \cdot 480 \text{ particles}.
\]

For the experiments performed in the laboratory a less powerful laser is used than the proposed 25 mW laser. The laboratory laser\(^5\) emits a power of 3.67 mW at a wavelength of 632 nm. The total power \( P_{\text{Mie}} \) reaching the sensitive area of the CCD chip is given as

\[
P_{\text{Mie}} = N_p \cdot I \cdot \frac{A}{r^2} \sum_{i=1}^{N_p} C_{\text{ext}}(a_i) \cdot p(\theta_i, a_i), \tag{5.2}
\]

where the incident irradiance \( I \) is 4893 W/m\(^2\). Thus, from Mie theory the value for the total power (in W) reaching the sensitive area of the CCD is given as (5.2). This value will be compared to the actual power reaching the CCD and used as a means of calibrating the response of the camera.

The concentration of particles is increased by adding an increasing number of latex microspheres to the 1500 ml container and transferring this suspension to the beaker in the setup. For every particle concentration an image is taken similar to the one shown in Figure 5.2, left. When the integrated signal in the CCD is read out it is converted into 16 bit giving each pixel a maximum value of 65535. The dark current is subtracted from the image and the entire pixel array is summed. This value \( E_{\text{CCD}} \) represents the total amount of scattered light reaching the CCD. The CCD chip has a full well capacity of 70,000 electrons. If a pixel is saturated it is read out as having a value of 65535 due to

\(^5\)The laser has been calibrated by the manufacturer, Edmund Optics, Barrington, NJ, USA
the 16 bit conversion. To get the number of electrons occupying the potential well of the pixel we must therefore multiply the pixel value by 70000/65535. Each electron in the potential well came from a photon\(^6\) having the energy \(\frac{h \cdot c}{\lambda}\). Dividing this by the exposure time \(t_{\text{exp}}\) thus gives us the power. The total power \(P_{\text{CCD}}\) reaching the sensitive area of the CCD is thus given as

\[
P_{\text{CCD}} = E_{\text{CCD}} \cdot \frac{70000}{65535} \cdot \frac{1}{Q\text{E}} \cdot \frac{h \cdot c}{\lambda} \frac{1}{t_{\text{exp}}},
\]

where \(Q\text{E}\) is the quantum efficiency of the CCD chip. This value will be compared to the theoretical value \(P_{\text{Mie}}\).

Figure 5.16 shows the comparison of the theoretical power \(P_{\text{Mie}}\) to the actual power \(P_{\text{CCD}}\) reaching the CCD. As can be seen the actual detected power is less than expected from theoretical modeling. This means that photons are lost on their way from being emitted by the laser, entering the beaker, being scattered by the particles in the beaker, exiting the beaker, and being detected by the CCD. This is not a surprise as reflections occur every time a change in refractive index are met. These reflections decrease the total power in the laser beam in ways not modeled in our case. Accurate ray tracing through multiple media and optical systems is very complex. By multiplying the experimental data with a factor of 1.2 an excellent fit is produced with a \(\chi^2 = 0.003\).

In the above description the response of the CCD has been calibrated using laser light scattering by latex microspheres. The theoretical response is compared to the experimen-

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\(^6\)Having subtracted the dark current from the image the remaining electrons have been generated by incoming photons.
5.4. **CALIBRATION OF THE CCD RESPONSE**

tal value and a very good agreement exists when multiplying the experimental data by a calibration factor of 1.2. With the calibration of the CCD response performed we turn to analysis of archived ice cores from Central Greenland.
Chapter 6

Dust Detection in Archived Ice Cores

This chapter discusses the data acquisition and analysis performed on archived ice cores. The data described in this work were acquired during 4 trips of about one week duration each to the National Ice Core Laboratory (NICL), part of United States Geological Survey, Denver, Colorado. Another trip was made to the Department of Geophysics, part of the Niels Bohr Institute at the University of Copenhagen, Denmark. The Denver facility stores ice from US coring projects including the international coring projects at Vostok, East Antarctica and the US-led deep drilling project at Summit, Greenland. This drilling project is termed Greenland Ice Sheet Project 2 (GISP2). The storage in Copenhagen holds ice from the Danish-led projects at Summit, Greenland, called the Greenland Ice Core Project (GRIP), and from the North GRIP (NGRIP) project located about 325 km NW of Summit. Whenever GISP2 ice or Vostok ice is mentioned the data acquisition took place at NICL and whenever GRIP/NGRIP ice is mentioned the data acquisition took place at the Department of Geophysics in Copenhagen.

As stated in the introductory remarks the design of the instrument is not optimized for detection of dust in archived ice cores. The driving idea behind the instrument is the desire for in-situ detection of dust and dust stratification from probes in ice sheets. As the means of field testing the Laser Dust Detector, namely the field deployment of the Cryobot, couldn’t be fulfilled, the main methods of testing and optimizing the performance of the instrument were to interrogate archived ice cores.

6.1 Data Acquisition

The light scattering data for the archived ice cores have all been acquired via experimental setups similar to the one shown in Figure 6.1. Most ice cores have been analyzed before by
CHAPTER 6. DUST DETECTION IN ARCHIVED ICE CORES

Figure 6.1: A sketch of the experimental setup used in acquiring digital images of archived ice cores. The length of the ice trough and linear translation stage is not to scale; all other dimensions are. The refraction of the laser beam as it enters (and exits) the ice is not shown in the figure.

various investigators and in general the ice cores have been cut leaving the cross section of the remaining ice shaped like a D. In our experimental setup the ice is supported by an acrylic trough throughout its entire length, with the flat surface of the ice positioned normal to the optical axis of the camera. The position and angle of the laser beam with respect to the optical axis of the camera is fixed and the laser beam is contained within a horizontal plane that also includes the optical axis of the camera. The laser penetrates the flat surface of the ice just outside the field of view of the camera and exits the ice outside the field of view, see Figure 6.2. This way light from reflections from the front and backside of the ice core should be outside the camera field of view, assuming a very smooth ice core surface generating a specularly reflected beam. A more rough surface will produce a diffusely reflected beam that, on the front side of the ice, tends to give a rather undefined laser incidence angle into the ice as well as diverging the beam. On the backside of the ice, the diffusely reflected beam tends to reflect light back into the ice; that reflection introduces an unknown amount of light with unknown characteristics into the image. Due to the unknown nature of this light it will act as noise in the image. The
Figure 6.2: A sketch showing the laser beam penetrating an ice core with dust stratification. The resulting 752x580 pixel image is produced by scattering of the laser light by the embedded dust. As the pencil-beam laser light penetrates the ice only a narrow stripe of scattered light is detected by the camera (assuming a single scattering environment). In the figure the ice core is represented by the negative of an actual linescan image of an ice core.

Laser light will penetrate the ice and cross the field of view of the camera going from right to left. The beam will be scattered by dust particles embedded in the ice and will, in an ideal situation, produce the 752x580 pixel image shown in Figure 6.2. The dark stripe at the far right in the acquired image is from scattering of the beam by a more dense layer of dust. The laser beam penetrates this dense layer of dust and scattering occurs close to the front side of the ice and hence close to the camera. The double layer at the far left in the image comes from light scattering close to the backside of the ice and hence at a larger distance from the camera. This change in distance gives a different scattering pattern for the two layers. Even if the two layers in the ice that generated the features at the far left and far right in the image were identical, the two scattering patterns in the image would be different. This difference is mainly due to a change in resolution when the distance is changed and a change in the scattering angle (as discussed in chapter 4).
The change in resolution will be discussed in detail in section 6.2 below.

The distances from the ice surface to the camera lens and laser head can be finely adjusted with a micrometer screw on a translation stage and the incidence angle and polarization of the laser can be changed by rotating the laser around a vertical and horizontal axis, respectively. Movement of the camera and laser parallel to the ice core is done with a computer controlled stepper motor connected to a linear translation stage. The camera is a digital astronomical camera\footnote{The camera is a Starlight XPress, MX716.} with a 752x580 pixel CCD array. It has a manual focus and diaphragm; the exposure time and image acquisition are computer controlled. At the beginning of the ice core interrogation the camera is positioned at the shallow end of the ice core, i.e. the end closest to the surface of the ice sheet at the time of coring. After the first image is taken the carriage holding the camera and laser system is translated a fixed distance along the ice core, representing an increase in depth. The translated distance is chosen to ensure enough overlap between the acquired images. Figure 6.3 shows a sketch of the image acquisition routine. The overlap of neighboring

Figure 6.3: A sketch of the image acquisition routine. The fixed translation between images represents about a third of the width of the field of view of the camera. The ice core in the sketch is represented by an actual scan of an ice core.
images is about a third of the width of the field of view of the camera. This means that a
given feature in the ice can be observed through 3 successive images. An example of this
can be seen as the dark vertical stripe, moving across the 3 first images in Figure 6.3. In
the first image the dark stripe, positioned at the far right in the image, is produced when
the laser encounters a dense layer of dust near the front surface of the ice. In the second
image the same dark stripe is produced when the laser light is scattered by the dense
layer positioned in the middle of the ice, and in the third image the dark stripe comes
from scattering of light close to the back side of the ice. Thus, the same feature in the
ice is imaged at three different distances which changes the resolution of the feature. The
change in resolution due to the change in distance takes some manipulations to account
for. This will be discussed in detail in the following section.

6.2 Correction of the Viewing Geometry

Laser light scattering experiments on small particles are usually done with a 90° scattering
angle between the laser beam and the optical axis of the detector. This makes data
analysis far easier than in a situation with non-perpendicularity, and reduces the impact
of small variations in geometry. Because the basis for the experimental setup in this
work is the integration of the Laser Dust Detector into the Cryobot, the data acquisition
cannot utilize a 90° scattering geometry. This necessitates some manipulations of the
images. These manipulations will be discussed in the following.

Consider an image of a metric calibration target, see Figure 6.4 (left). This calibration
target\(^2\) consists of a number of black lines equidistantly spaced 1 mm apart, with a high
degree of accuracy. The face of the calibration target is normal to the optical axis of
the camera. The linear field of view (LFOV) of the optical system (the lens system and
CCD camera) is then defined as the number of mm-lines one is able to see in the image.
In Figure 6.4 (left) the LFOV is 28 mm where the face of the target is normal to the
optical axis of the camera. As shown in Figure 6.2 and 6.3, the laser beam does not cross
the field of view of the camera at an angle normal to the optical axis. The path of the
laser beam is oblique to the optical axis and therefore an image of the laser beam would
more resemble the situation in Figure 6.4 (right) where the same calibration target has
been imaged but now at the same oblique angle as the laser beam crossing the field of
view of the camera in Figure 6.2. Looking at the rotated target it is evident that the
LFOV increases when turning the calibration target at an angle to the optical axis of the
camera compared to the perpendicular geometry in Figure 6.4 (left). Next it is evident

\(^2\)The calibration target is the "ES Magnifier Quality Resolution Chart" from Edmund Scientific Com-
pany, Barrington, New Jersey, USA.
that the mm-lines are compressed at the distant side (the left side of the target) and expanded at the closer side (the right side of the target). This compression and expansion of mm-lines on the calibration target illustrates the need to transform the images to a situation as if they were acquired in a perpendicular geometry. The reason for this is the following: Consider a situation where we have acquired an image of dust layers embedded in ice. The data set of interest is the stratigraphy of the ice. As a consequence of the oblique laser path though the field of view of the camera the layers will be imaged as the calibration target in Figure 6.4 (right), and therefore these layers will not display the true stratigraphic spacing. The layers would show up compressed at one side and expanded at the other side like the mm-lines on the rotated calibration target. Therefore, corrections of the digital images will be part of the data processing required in analyzing the images.

6.2.1 The Scaling Function $S(c)$

When transforming an image from the oblique viewing geometry to a perpendicular viewing geometry there are two effects that need to be accounted for. Looking at Figure 6.4 (right) we realize that one effect is due to the change in distance from the camera to the individual lines on the calibration target. Below we define a function $S_d$ (a 752 element vector) that all the acquired images are multiplied by (column by column) to stretch and compress the individual image columns to account for the change in distance.

Another effect is due to the change in angle between the face of the calibration target and the line of sight across the target. This effect is named the sine effect $S_s$ (also a 752 element vector) and is discussed below. The combined effect is then given as the product of the distance and the sine effects. The appropriate scaling function, that the acquired
6.2. **CORRECTION OF THE VIEWING GEOMETRY**

images will be multiplied by, is defined as \( S = S_d \cdot S_s \) (multiplied element by element).

The acquired images are 752x580 pixel arrays. We use, for this example, a focus distance of 50 mm. That is, if we had a perpendicular viewing geometry, the image would be at a distance of 50 mm. The transformation of the oblique image will then be done by taking one of the 752 columns at a time. Starting at the first column (the left edge of the image) the distance to the corresponding object point in space is found. In the case of the calibration target this would be the left most mm-line. If the object point is further away than 50 mm the column will be expanded, if the object point is closer than 50 mm the column will be compressed. On top of this the sine effect will be applied to every column. This is done for every column in the image resulting in an image that is transformed from the oblique viewing geometry to the perpendicular viewing geometry.

**Determining Line Separation vs. Distance**

One reason for the compression and expansion of the mm-lines on the calibration target in Figure 6.4 (right) is that the distances to the individual mm-lines are changed, see Figure 6.5. The angular field of view (AFOV) of the camera is named \( \phi \) in the figure. The laser beam crosses the field of view at an oblique angle \( \alpha \) with respect to the perpendicular laser geometry. The focus distance \( f \) is the distance from the lens of the camera system to the object plane. There is one distance that has the best focus and a range in distance exists, where the focus is essentially unchanged, called the depth of field of the camera.
system. The camera system used in acquiring the data for this work has a depth of field large enough for the entire oblique laser beam to be in focus.\(^3\)

In order to transform the oblique images into images with a normal viewing geometry we need to find the distance from the camera to every point along the oblique laser beam crossing the field of view. This distance then determines whether we need to expand or compress that part of the image in order for the image to be transformed into perpendicular viewing geometry. The distance to the camera is given by the length of line \(c\) in Figure 6.5. Using the solid lines \(a\), \(b\), and \(c\) and the angle \(\gamma\), the cosine relation gives

\[
c^2 = a^2 + b^2 - 2ab \cos \gamma,
\]

where angle \(\gamma\) is given by

\[
\gamma = 180^\circ - \phi/2 - (\alpha + 90^\circ) = 90^\circ - \phi/2 - \alpha.
\]

Having found the distance to every point along the laser beam (or calibration target) we need to find the specific amount of compression or expansion that is applied to the image columns, expressed as a function of the distance \(c\).

In the following we experimentally determine the separation of the mm-lines in a set of images of the calibration target as a function of the distance to the target. This is then used as the scale for changing the column width. A series of images of the calibration target, imaged in normal viewing geometry as in Figure 6.4 (left), is taken with increasing distance to the target. The focus is fixed at a midpoint value in distance and the total change in distance to the target is always smaller than the depth of field, i.e. the images are always in focus. We want to find the separation between the mm-lines as a function of the distance from the camera (the focus distance). The further away the calibration target is the smaller the line separation will be. For every distance from the camera a profile of the alternating black and white lines is made similar to the one seen in Figure 6.6 (left). The position of the peaks (X) and troughs (squares) occur where the derivative of the profile is zero. This occurs in the middle of the white and black stripes, respectively. The line separation is then found by measuring the distance between two successive peaks or two successive troughs (X’s or squares). For every focus distance the mm-line separation is given as the relative line separation for the middle quarter of the image. The middle quarter of the image represents the true normal distance to the camera.

In Figure 6.6 (right) the line separation is plotted as a function of the distance to the camera. The line separation and focus distance are given relative to that at a focal distance of 50 mm, which is the setting of the camera used. A 2nd degree polynomial least squares fit \((0.05c^2 - 0.36c + 1.31)\) has been made to the data with \(\chi^2 = 3.0 \times 10^{-5}\).

\(^3\)For a more complete description of optical systems see e.g. [Hecht, 2002]
6.2. CORRECTION OF THE VIEWING GEOMETRY

Figure 6.6: Left: A 'horizontal' profile across an image of the calibration target. The midpoints of the white lines are marked with an 'X', the midpoints of all black lines are marked with a square. Right: The line separation plotted as a function of the distance to the camera. The line separation and the distance to the camera are both given relative to the line separation and distance at the focal distance of 50 mm used in the setup. The dotted line is a fit to the data.

This polynomial fit (shown in Figure 6.6 (right)) thus shows, that if the distance to the target is halved (relative distance = 0.5) the line separation is increased by a factor of 1.14 (relative line separation = 1.14).

For every column in a given image the distance \( c \) to the corresponding object point in space is found. As the images have 752 columns, \( c \) is a 752 element vector. The line separation for the calibration target is now given as \( 0.05c^2 - 0.36c + 1.31 \), with the distance \( c \) inserted. If the viewing geometry was perpendicular the line separation would have been constant (1.0) as the distance to every mm-line would have been constant (50 mm in this case). To counteract the effect of the oblique geometry the distance part \( S_d(c) \) will compress or expand every image column, with

\[
S_d(c) = (0.05c^2 - 0.36c + 1.31)^{-1}.
\]

Determining the Sine Effect

The other part of the scaling function is a sine term \( S_s(c) \). When you look across an oblique image from the left to the right edge, see Figure 6.5, not only does the distance to the laser beam change but also the angle \( \beta \) changes. At the left edge of the image the Oblique Laser Geometry line is seen more edge on \( (\beta \rightarrow 180^\circ - \gamma) \) and at the right edge of the image the Oblique Laser Geometry line is seen almost face on \( (\beta \sim 90^\circ) \). This results in a change in the perceived separation of the mm-lines which is accounted for by
multiplying by the inverse of \( \sin \beta \), where

\[
S_s(c) = (\sin \beta)^{-1} = \frac{c}{b \cdot \sin \gamma}.
\]

This gives us the full scaling function \( S(c) \) as

\[
S(c) = S_d(c) \cdot S_s(c) = (0.05c^2 - 0.36c + 1.31)^{-1} \cdot \frac{c}{b \cdot \sin \gamma}.
\]

Figure 6.7 shows a plot of the scaling function vs. the image column. That is, the first

![Figure 6.7: The scaling function \( S(c) \) used in transforming images seen in the oblique viewing geometry to images viewed in perpendicular geometry.](image)

column should be expanded by a factor of 2.27 whereas the 752nd column should be expanded by a factor of 1.1. How the transformation is performed in reality is described below.

6.2.2 Transformation of Images

This section describes in detail how an image that is viewed in the oblique viewing geometry is transformed into an image as it would have appeared if viewed in the perpendicular viewing geometry. Going through all the image columns one by one, the first column should be expanded by a factor of 2.27 according to the scaling function \( S(c) \). As a pixel array obviously consists of an integer number of rows and columns, the first column should be enlarged to 2 columns (the round off of 2.27). To get a better result the columns of the oblique image is first enlarged 100 times by bilinear interpolation. This gives a 75200x580
6.2. CORRECTION OF THE VIEWING GEOMETRY

pixel array. The first 100 columns are then expanded to 227 columns by linear interpolation. The columns in the oblique image is then expanded in groups of 100 until the last 100 columns is expanded into 110 columns, again according to the scaling function.

All the columns are expanded by a factor of more than one, which is due to the sine part of the scaling function. This way the transformed image is a much bigger pixel array than the oblique image. Therefore the transformed image is finally shrunk into a 752x580 pixel array, giving us the desired transformed image. The shrinking of the image is done via a cubic convolution interpolation method (performed in IDL⁴). Figure 6.8 (left) shows the product of the transformation on the image in Figure 6.4 (right). As can be seen

Figure 6.8: Left: An image of the calibration target acquired in the oblique geometry. The transformation routine has been applied to the image and it appears as if it was acquired in the perpendicular viewing geometry. Right: The plot shows the line separation for the oblique (black) and perpendicular (red) images, respectively. The straight horizontal line is evidence of an equidistant line separation in the transformed image.

the image appears to be acquired in the perpendicular viewing geometry and the mm-lines are spaced equidistantly. A way to test the transformation routine is to measure the separation between the individual mm-lines in a single image. As mentioned, the line separation changes across the image if it is obliquely viewed and shows a constant line separation if the image is viewed in the perpendicular geometry. Figure 6.8 (right) shows the line separation for the original oblique image (black data points) and for the transformed image (red data points). A 2nd degree polynomial fit has been made to both data sets. The transformation routine thus establishes an equidistant line separation across the image, as is evident by the straight horizontal fit to the data points. When

⁴Interactive Data Language, Research Systems, Inc.
image acquisition of the archived ice cores is done, the transformation routine is applied to every image before further analysis is performed.

6.3 Correlation of Images

This section describes how to 'stitch' neighboring images together to get a continuous profile of the ice core. As described above the ice core is interrogated by taking a series of images along the ice core with a given overlap between neighboring images. One of the analyses performed on the acquired images is to make a profile of the light intensity as a function of depth. To make a continuous profile of the ice core the transformed images have to be stitched together. The camera used in the experimental setup has an angular field of view of 31.3°. At a focus distance of 50 mm this gives a linear field of view of 28 mm. The CCD chip is a 752x580 pixel array giving a 'horizontal' resolution of 28 mm/752 pixels = 37 µm/pixel. The linear translation stage that moves the camera and laser system along the ice core is computer controlled and can be set to translate a given distance with a high accuracy. To be able to stitch two neighboring images together on a pixel to pixel basis the resolution of the translation stage must be better than 37 µm. The pitch of the axel on the translation stage is 5 mm/revolution. The stepper motor that drives the axel has 4000 counts/revolution, thus the resolution of the translation stage is 5 mm/4000 counts = 1.27 µm/count. This resolution is far better than the resolution obtained by the camera and the translation stage is thus able to position the camera at the given position with a subpixel accuracy.

Although the translation stage in theory is able to position the camera within an image pixel, that is not always the case in reality. If, for example, the ice core isn’t setup perfectly horizontally the same feature in two neighboring images can be shifted more than 37 µm (in this case the shift will be vertical) and thereby be shifted more than a pixel. We therefore need a procedure to automatically search for the same feature in two neighboring images and align the images accordingly. To do this we have utilized the so called cross correlation function that is a standard method of estimating the degree to which two data series are correlated. The function $P_{xy}(L)$ computes the cross correlation
of two sample populations $x$ and $y$ as a function of the lag $L$, of two sample populations $x$ and $y$ as a function of the lag $L$,

$$P_{xy}(L) = \begin{cases} 
\frac{\sum_{i=0}^{N-|L|-1} (x_{i-|L|} - \bar{x})(y_i - \bar{y})}{\sqrt{\left[\sum_{i=0}^{N-1} (x_i - \bar{x})^2\right]\left[\sum_{i=0}^{N-1} (y_i - \bar{y})^2\right]}}, & \text{for } L < 0 \\
\frac{\sum_{i=0}^{N-L-1} (x_i - \bar{x})(y_{i+L} - \bar{y})}{\sqrt{\left[\sum_{i=0}^{N-1} (x_i - \bar{x})^2\right]\left[\sum_{i=0}^{N-1} (y_i - \bar{y})^2\right]}}, & \text{for } L \geq 0 
\end{cases}$$ (6.2)

where $\bar{x}$ and $\bar{y}$ are the means of the sample populations $x = (x_0, x_1, x_2, ..., x_{N-1})$ and $y = (y_0, y_1, y_2, ..., y_{N-1})$, respectively.

Figure 6.9 shows two negative neighboring images with an overlap corresponding to about 1/2 the horizontal image size. The right image is the 'deep' image meaning that is an image taken of a deeper part of the core than the left image. The dark stripe in the images, representing a more dense layer of dust, moves to the left as the camera moves down the core (towards the deeper end of the core). In the deep image a correlation area is marked that is used to correlate the two images. The area is chosen to be rectangular with a larger horizontal side than vertical side. As the images consist of alternating dark and bright stripes, pattern recognition is more efficient with a horizontal rectangular area instead of say, a square area. Assuming a 'perfect' translation of the camera for a given

![Figure 6.9: Two neighboring images showing a translation of about 1/2 the image width. The blue 'best match' area in the left image is found to match the red 'correlation' area in the right image by computing the cross correlation function of the two areas. The shallow image is shown to the left and the deep image is shown to the right.](image-url)
distance, the identical area (here called the test area) is marked as the small red rectangle in the shallow image to the left. Knowing that the translation might not be perfect we need to search for the identical area in the neighborhood of the proposed test area. The search area is marked by the large red rectangle. The cross correlation function of the 'correlation area' and the 'test area' is computed for all the possible positions of the test area within the search area. For every computation of the cross correlation of the two areas the result is stored in a matrix. This matrix is shown as the surface in Figure 6.10 (left). Thus, every point on the surface in Figure 6.10 (left) represent the cross correlation for that specific position of the test area within the search area. The best match is found at the position of the peak and is represented by the blue rectangle in Figure 6.9 (left).

The shape of the surface in Figure 6.10 (left) can be explained as follows: The test area starts at the upper left hand corner of the search area. The cross correlation of this area and the 'correlation area' in the deep image is computed. This is done for every position of the test area as it steps through the columns of the search area and reaches the top right hand corner. This data series represents the first row in the cross correlation matrix that is shown in Figure 6.10 (left). For the first row the best correlation is found when the two dark stripes are aligned. Stepping through all the rows of the search area one specific row gives the best correlation which is represented by the peak on the surface. The cross correlation surface shown in Figure 6.10 (right) represents the correlation between two more irregular images as the ones shown in Figure 6.14 in the following section where different geometrical setups are discussed.

For every pair of neighboring images the same method is used to automatically find
the best position for overlapping the images and stitching them together.

6.4 Image Analysis of Archived Ice Cores

This section describes the analysis of images acquired via laser light interrogation of archived ice cores. The instrument designed in this work is optimized for in-situ detection of embedded scatterers in ice sheet ice. Therefore the image analysis performed in this section is intended as a means of verifying the performance of the Laser Dust Detector and is thus not intended as an analysis of the nature or characteristics of the ice or embedded dust.

NGRIP Ice

During a 2.5 month stay at the Department of Geophysics at the University of Copenhagen, Denmark, the performance of the Laser Dust Detector was analyzed via interrogations of archived ice cores from NGRIP. This ice is rich in paleoclimatic information and has regions of dense stratification of dust. The Department of Geophysics have been at the forefront of experimental analysis of archived ice cores for decades and has vast experience in designing methods for ice core interrogation and analysis. As optical interrogation of archived ice cores is a quite common technique for non-destructive analysis of embedded microparticles, the ice had previously been optically interrogated with a linescanning device. This device uses a broad spectrum indirect light source to illuminate the interior of the ice. A thin slab of the ice was cut through most of the extracted ice core and imaged via a narrow slit digital camera [Svensson, 2003]. This technique produced a continuous linescan of the examined core. A small part of this linescan can be seen in Figure 6.11 where a plot of the light intensity of the linescan has been included. The negative linescan is shown for illustration purposes. The profile is that for the ‘positive’ linescan, i.e. a peak in the intensity profile illustrates more light being scattered in the direction of the camera and hence indicates a more dense layer of scatterers in the ice. The resolution of the linescan is 12 pixels per mm which makes the length of the ice core represented by the linescan in Figure 6.11 about 103 mm.

The general setup of the experiment is shown in Figure 6.1 above. Different versions of this setup were used however, to minimize the amount of noise in the acquired images. Some of these version are discussed below. As mentioned earlier the design of the Laser Dust Detector described in this work is not optimized for interrogation of archived ice cores and the best performance of the instrument cannot be demonstrated. The main reason for this is the small dimensions and corresponding surface scattering of the ice cores being analyzed. A great deal of effort has been made in order to minimize unwanted
CHAPTER 6. DUST DETECTION IN ARCHIVED ICE CORES

Figure 6.11: A linescan of a small part of the NGRIP ice core at 1828.75 m depth. A profile of the light intensity is shown below.

reflections from the surfaces of the ice core. Part of this effort has been to adjust the geometry of the camera and laser system. A 45° fan-laser has been used in some of the setups. By mounting the laser above the camera lens and directing the fanned-out beam vertically down on the ice some of the noise is reduced. This setup can be seen in Figure 6.12. The setup has been used to analyze a small part of the NGRIP ice core at the depth 1828.75 m - 1829.30 m. By using the vertical fan-laser diffuse reflections from the backside of the ice core are reduced compared to the original horizontal pencil-beam laser sketched in Figure 6.2. Using the vertical fan-laser the viewing geometry, as discussed in section 6.2, is near perpendicular. The drawback of this method is the reduction of laser irradiance as the fan beam is produced by a lens system in front of the laser aperture. The same light power is output but the irradiance decreases as the light is spread out.

Another adjustment of the setup can be seen in Figure 6.13. When the dimensions of the interrogated ice core are small the diffuse reflections off the backside of the ice will take place inside the field of view of the camera. The reason for this is that the laser incidence angle at the air-ice interface cannot be made large enough. This is illustrated in Figure 6.13 (left.) The figure illustrates Snells Law for different incidence angles from 15° to 90° in increments of 15°. As the incidence angle $\theta_i \rightarrow 90°$ the transmittance angle...
Figure 6.12: A side view of the fan-laser setup. The plane of the fanned-out laser beam is viewed edge-on. The two different orientations of the ice core produces different refractions of the incident fan-laser beam.

\[ \theta_t \rightarrow 49.8^\circ. \] An experimental setup for incidence angles above 49.8° is shown in Figure 6.13 (right). Small notches have been cut in the ice to make the laser beam penetrate the air-
ice interface at 90°. By shaping the notch the transmittance angle can have any desired value. Some of the drawbacks of this setup are that it is a destructive technique for the ice and that the images are somewhat disturbed in the notch region itself. Another drawback is that the laser is fixed with respect to the ice core and not with respect to the camera. When the camera is translated a given distance (to the left in Figure 6.13 (right)) the laser light is attenuated (assuming the existence of scatterers in the ice). This makes direct comparison between neighboring images not straightforward.

Figure 6.14 shows four negative 'scans' of a small part of the NGRIP core. Four different setups have been used in acquiring the scans. Scan A is the linescan mentioned above, scan B and C are the setups showed in Figure 6.12 (left) and (right) respectively, and scan D is the setup shown in Figure 6.13 (right). The three lower scans have been stitched together from 20 images using the cross correlation procedure described in the previous section. Stitching two neighboring images together produces a typical cross correlation surface as the one shown in Figure 6.10 (right) above. All the scans are negative images, i.e. the dark bands represents more light scattered into the camera and thus indicate a more dense layer of scatterers.

The linescan (scan A in Figure 6.14) is acquired under optimal conditions [Svensson, 2003]. It is made in a 90° scattering geometry where a thin slab of ice has been cut through the entire length of the ice core and illuminated from the sides. The light used to illuminate the interior of the ice is produced from a broad spectrum indirect light source. This way unwanted reflections inside the ice are minimized and only light scattered by impurities or air bubbles is detected by the camera. While the performance of the Laser Dust Detector isn’t optimized for interrogation of archived ice cores the results of scan
B, C, and D are nevertheless excellent. The visual resemblance of scan B and C to the linescan is at first glance better than scan D. This is due to the nature of the experimental setups. Scan D is made with a pencil-beam laser and is therefore restricted to scatter light in a narrow vertical region. As the laser is fixed with respect to the ice, and not the camera as is the case for scan B and C, the divergence of the laser beam is noticeable, especially when the camera is translated away from the laser a number of images. In the middle of scan D a new notch is encountered that gives the abrupt change in light intensity. To the far right in scan D another notch is made. To the left of the notches the scattered patterns resemble scan A in great detail. The divergence of the beam over the next 10 images (the 'distance' between two notches) makes the pattern look fuzzy.

The comparison of scans B, C, and D to the linescan in Figure 6.14 is, by visual inspection, found to be very satisfactory as all the major dust bands (dark stripes) and regions of clean ice (light bands) are included in scans B, C, and D. Another way to test the performance of the Laser Dust Detector is to analyze the light intensity profile of each scan. In Figure 6.15 - 6.17 the profiles of scans B, C, and D are plotted against that of the linescan.

Figure 6.15: The light intensity profile of the fan-laser scan (scan B) and the linescan (scan A).

The light intensity profiles have been made by averaging over 290 rows, i.e. the middle half of each scan. To reduce the high frequency oscillations a standard digital low-pass filter has been applied in the following way. First a Fourier transform has been applied to the profile. Then a low-pass filter with a given cutoff has been applied to the Fourier transform to zero all ”frequencies” above the cutoff, and last the inverse Fourier transform has been applied to the filtered function to give the smoothed profiles seen in Figures 6.15 - 6.17. A 12 bit scaling of the scans B, C, and D has been applied as the linescan has been saved as 12 bit, whereas the digital images acquired with the Laser Dust Detector are 16 bit.

All three profiles compare well to the linescan profile. More high frequency oscillations
are seen in the B, C, and D profiles than in the linescan profile. This is partly due to the fact that the linescan has a resolution of 12 pixels/mm where the other scans have a resolution of 33 pixels/mm. The higher resolution for the B, C, and D scans make the profiles oscillate more. The other reason is the higher amount of noise in the images acquired with the Laser Dust Detector. All this being said, the major peaks and troughs in the linescan light intensity profile are closely followed in the B, C, and D profiles, concluding that even though the instrument isn’t optimized for this type of analysis on archived ice cores the performance is very satisfactory.

GISP2 Silty Ice

In the previous section ice from the NGRIP drill site on Greenland was interrogated as a means of testing the performance of the Laser Dust Detector. The scattering of laser light by dust in the ice cores was assumed to take place in the single scattering domain. This is a valid assumption in most of the bubble free ice from Greenland, except in the last few meters, close to the bedrock, where the ice is mixed with a large amount
of dust and debris. In this silty ice the single scattering assumption breaks down, but simple analysis of the ice is nevertheless possible. A chronological stratification of dust layers in the silty ice doesn’t exist as the ice is heavily disturbed by the flow over the bedrock and the large amount of dust and silt. However, the ice can be used as possible analogue of Martian ice as it is expected that some Martian ice contains a large amount of dust. Figure 6.18 shows an image of the GISP2 ice core at 3040.34 m depth. The ice is completely opaque with a band of semi-transparent ice, marked by the line d in the figure. Small conglomerates of dust and sand have been cut and are exposed at the ice surface. The largest of these are marked by the letters a, b, c, and e in Figure 6.18. Quantitative dust profiling is very difficult to do as the path length of the laser light inside the ice is limited due to heavy attenuation of the beam. Simple light scattering can be done as can be seen in Figure 6.19. Features marked by letters correspond to the same features as in

Figure 6.18: The GISP2 ice core near the bedrock (3040.34 m depth). The conglomerate marked ’a’ is 4x5 mm in size.

Figure 6.19: A laser light interrogation of the silty GISP2 ice.

Figure 6.18. The conglomerates a, b, c, and e appear dark as they block the scattered light from entering the camera. The dark area marked d appears dark as the amount of dust here is much smaller than in the rest of the silty ice. Compared to the rest of the GISP2 ice core it is still very dusty but in contrast to the opaque ice it is relatively clean.
ice. This way dust stratification can still be analyzed in a simple way as long as enough contrast exists in the ice. Figure 6.20 is an image of the same piece of ice taken with an external LED light source mounted on the camera. This illustrates the experimental method one would utilize in the situation where the ice is completely opaque and light only penetrates the outer few mm to cm of the ice. A detailed view of the outer surface of the conglomerates can be seen. By combining the image acquisition methods used in Figure 6.19 and 6.20 ice with a wide range in dust load can be qualitatively analyzed.

Figure 6.20: An image of the silty GISP2 light acquired with an external LED light source.
Chapter 7

Conclusion

The JPL Cryobot, designed at the Jet Propulsion Laboratory, Pasadena, California, USA, is a tethered robotic vehicle that is designed to investigate planetary ice sheets by melt penetration. The Cryobot was designed, built, and field tested at the Norwegian island of Svalbard, during the years 1999-2002. It is a modified version of the Philberth probe [Philberth, 1976], in that it incorporates a miniature hot-water drill to increase the melting rate as well as providing the capability of penetrating through layers of dust and debris. Having the desired capability of penetrating kilometers of ice sheet ice, the Cryobot is an excellent instrument platform for in-situ detection and analysis.

The primary purpose of this work is to design an instrument, the Laser Dust Detector (LDD), that is able to measure, from an in-situ probe, the vertical profile of scatterers in the local environment of an ice sheet. The goal is to have the instrument integrated into the hull of the ice penetrating Cryobot. The Cryobot would be the instrument platform from which the LDD would be deployed, and therefore a field deployed test Cryobot would be an important tool in designing and optimizing the performance of the instrument. Unfortunately the completion of the Cryobot was postponed, and another method of verifying the design of the LDD had to be utilized. Due to the availability and strong analogs, archived ice cores were chosen as the subject of testing the performance of the instrument, emphasizing that the design geometry would still be evaluated for an in-situ study and not a study performed in the laboratory. As the analysis of archived ice cores was not the objective of the instrument, complete and final design of the Laser Dust Detector is not achieved. However, an instrument has been designed that is able to detect impurities in archived ice cores on the same scale as the conventional methods of non-destructive analysis of embedded dust in ice, namely the 90° light scattering technique. Numerous investigators have analyzed embedded dust in archived ice cores with a range of interrogation techniques, see section 3.1, but none has approached the technique of fine resolution (< 1 mm) in-situ detection of dust in ice sheet ice.
A rigorous analysis of the possible geometries of the Laser Dust Detector has been performed, see section 3.2. This has been necessary as a strict 90° scattering geometry couldn’t be applied due to various geometrical constraints. As the method of dust detection is based upon detection of light scattered by impurities in the ice, controlling the laser light emitted through the hull of the Cryobot is of great importance. The laser beam is directed through the hull via a fiber optics cable and encounters different media before entering the ice, see section 3.2. Going through the interfaces between the different media results in a change in the refractive index encountered by the laser beam. An important part of optimizing the Laser Dust Detector is to find the optimal range in scattering angles \([\psi_1, \psi_2]\) (see Figure 3.5) for which the camera detects scattered light, under the constraint that the irradiance should be uniform throughout the range \([\psi_1, \psi_2]\). As the geometry of the camera-laser system allows the laser beam to have a transmittance angle into the ice \(\theta_{\text{ice}}\) in the range \([24.9°, 69.3°]\) the ranges in the two scattering angles become \(\psi_1(\theta_{\text{ice}}) \in [78.1°, 141.4°]\) and \(\psi_2(\theta_{\text{ice}}) \in [101.9°, 165.2°]\). The camera used in this work has an effective angular field of view of 23.8°. Therefore a subset of scattering angles, equal to 23.8°, must be chosen from these two ranges. Which subset to choose is determined by the pattern of scattered light received by the camera, governed by the theory of Mie scattering.

A series of computer simulated scattering patterns have been made for the case of a single spherical particle and for a size distribution of spherical particles similar to typical distributions found in archived ice cores from Greenland, see section 4.2. This way a scattering pattern has been made that allows us to chose the range in scattering angles \([\psi_1, \psi_2] = [100.0°, 123.8°]\) as this range has the least change in irradiance over a 23.8° range in scattering angle, see Figure 4.11. The chosen range in scattering angle corresponds to an incidence angle of the laser beam in the hull of the Cryobot, equal to \(\theta_{\text{hull}} = 60.3°\).

An assumption of single scattering is made in the computer modeling of Mie scattering on spherical particles as well as in the laboratory experiments performed in this work. This assumption depends on the concentration of scatterers in the investigated sample. In laboratory experiments it thus depends on the concentration of latex microparticles in a suspension of water or it depends on the concentration of micron-sized particles in an archived ice core. As van de Hulst [van de Hulst, 1957] points out a linear relationship between the scattered signal and the concentration of scatterers must exist in the single scattering regime. An experiment based on this concept is performed that justifies the assumption of single scattering for Greenland ice sheet ice by estimating the upper limit for the concentration of microparticles allowed in the single scattering domain, see section 5.1. To find the single-scattering limit laser light was directed into a beaker filled with a suspension of microparticles at a precisely known concentration; the scattered light was
then measured as a function of an increasing particle concentration. As suspected the relationship between the scattered light and the particle concentration is at first linear indicating single scattering. As more and more particles are added, more photons are scattered more than once, the multiple scattering domain is reached, and an estimate for the single/multiple scattering transition is made. At a concentration of more than $3.5 \times 10^6$ particles/ml the single scattering assumption can no longer be justified and the upper limit for single scattering is thus set at that value. The GRIP ice cores investigated in this work have a maximum particle concentration of $6.5 \times 10^5$ particles/ml [Steffensen, 1997], see Appendix B. We note that planetary ice sheet ice have an unknown particle density.

The Laser Dust Detector is designed on the basis of Mie scattering of laser light on small particles. A laboratory experiment is performed that verifies that the Laser Dust Detector geometry is in agreement with the Mie model, see section 5.2. Laser light is directed into a beaker filled with a suspension of latex microparticles; the scattered irradiance is then measured as a function of the scattering angle (the angle between the incident laser beam and the optical axis of the camera). The observed scattering compares very well to a computer generated Mie scattering model performed on identical particles, see Figure 5.8, as well as a size distribution of particles, see Figure 5.10. An important parameter in this experiment is the homogeneity of the particle suspension. Assuming a homogeneous distribution of particles in the beaker at the beginning of the experiment the suspension time of the particles determines how long the suspension remains homogeneous. The settling velocity is calculated on basis of Stoke’s law to 0.43 mm/hour, thus justifying the assumption of homogeneity.

The optimal performance of the Laser Dust Detector is dependent on the number of photons reaching the sensitive area of the detector. An estimate is made of the number of photons needed in order to have a 'good' signal to noise ratio. This estimate is then compared to a calculation of the number of photons reaching the CCD for a number of parameters, including the power of the laser and realistic values for the exposure time of the camera, see section 5.3. The calculation of the number of photons reaching the sensitive area of the detector is based on the range in number density of particles typically found in archived ice cores from Greenland [Steffensen, 1997]. A gaussian size distribution has been applied to the particles even though this is not the distribution typically found in the ice. For a laser power of 25 mW the resulting exposure time of the camera is in the range [2.9 ms, 0.2 s]. The shorter exposure time corresponds to the maximum amount of dust particles found in the GRIP ice core (Central Greenland), and the longer exposure time to a typical value for the lower number of dust particles.

An important calibration of the Laser Dust Detector is to convert the scattered light
intensity received by the camera to the number of scatterers in the investigated sample. This is done (see section 5.4) for our experimental geometry, laser power and camera system. In an experimental setup similar to the one determining the limit for single scattering dust concentration, the amount of scattered light is measured as a function of particle concentration. A detailed calculation of the expected power reaching the CCD is compared to the experimental data and a very good fit is obtained by multiplying the experimental data by a factor of 1.2, see Figure 5.16. The multiplicative factor of 1.2 tells us that a small fraction of photons are being lost somewhere on the path from being emitted by the laser, entering the beaker containing the suspended particles, being scattered by the particles, exiting the beaker, and finally entering the camera and lens system. This is not unexpected at all as a loss free environment is experimentally very hard to obtain. Reflection of the laser beam at the interfaces air→glass and glass→water is part of the reason for the 'lost' photons.

The Laser Dust Detection cannot utilize a 90° scattering geometry, and this introduces constraints on the data acquisition routine and necessitates manipulations of the acquired images as part of the image analysis. The laser beam crosses the field of view of the camera in an oblique angle to the optical axis which introduces a spatial distortion of the acquired images. A routine has been made to undistort the images and a detailed description of this transformation can be found in section 6.2. When image acquisition of the archived ice cores is done, the transformation routine is applied to every image before analysis is performed.

A digital image of the entire length of the NGRIP ice core (a linescan) has been made by previous investigators [Svensson, 2003]. An important way to test the performance of the Laser Dust Detector is to compare a light intensity profile of the acquired images with a light intensity profile of the digital linescan taken of the same ice core. To do this a routine has been made to 'stitch' the individual LDD images together to get as precise an overlap of neighboring images as possible and thereby produce a similar linescan, see section 6.3. This stitching routine utilizes a cross correlation function that is a standard way of estimating the correlation of two data series. When the cross correlation routine has been applied to every pair of neighboring images the resulting image mosaic or linescan is ready to be analyzed to test the performance of the Laser Dust Detector. When Cryobot images are acquired, image transformation and stitching, as described here, will be required.

The image analysis performed in this work is intended to test the performance of the Laser Dust Detector; it is not an analysis of the characteristics of the ice core itself. Different geometrical setups have been tested to minimize unwanted noise and thereby optimize the performance of the instrument, see section 6.4. Three different light intensity
profiles have been made of the three most optimal geometries, see Figures 6.15 - 6.17. The comparison of these profiles with the linescan light intensity profile is very close and follows all the major peaks and troughs. This demonstrates an excellent performance of the Laser Dust Detector in finding the true stratigraphy of dust deposits in ice sheets.

For the purpose of testing the characteristics of the instrument and optimizing the design, artificially made ice would be a good agent. However, making completely bubble-free ice is a major task. It can probably be done, as has been demonstrated for ice free of large bubbles, but making a stratigraphic layering of embedded dust, in the bubble-free ice, becomes a challenging and very time consuming process. This task is beyond the scope of this work but it would be a possible way of determining the optimal design for the instrument, but not nearly as effective as deployment of the instrument in an ice sheet.

The goal of this work is to design and optimize an instrument for in-situ dust detection in the local environment of an ice sheet. The instrument, the Laser Dust Detector, has been designed and the performance is found to be adequate to the task of estimating profiles of particles in the Mie scattering domain (a few microns) in concentrations consistent with single scattering. In planetary exploration, particle concentration can now only be guessed at, but the atmospherically transported dust on Mars is known to be in the Mie size range. Consequently, the Laser Dust Detector is an excellent candidate for the in-situ exploration of icy cites in the Solar System.
Appendix A

Microsphere Specifications

A.1 The Number Density of Particles

In section 5.1 an experiment is discussed that determines the upper limit for the number density of particles in the single scattering domain. The number of particles per milliliter is determined in the following way.

The microparticles are supplied in a 2.6% aqueous solution in a dropper bottle. The supplier\textsuperscript{1} prepares the solution by adding 2.6 gram of polyurethane microparticles to 100 milliliter of ultra pure water. The particles have a mean radius of 0.75 µm with a very narrow Gaussian size distribution (standard deviation $\sigma = 0.02$ µm). The density is $\rho = 1050$ kg/m$^3$.

The mass $m$ of 1 particle is:

$$m = \rho \cdot V = 1050 \text{ kg/m}^3 \cdot \frac{4}{3} \pi (0.75 \times 10^{-6} \text{ m})^3 = 1.86 \times 10^{-15} \text{ kg},$$

where $V$ is the volume of the particle. The solution contains 0.026 gram of microspheres per milliliter. The number of particles in 0.026 gram is

$$\frac{0.026 \text{ g/ml}}{1.86 \times 10^{-12} \text{ g}} = 1.4 \times 10^{10} \text{ particles/ml}.$$

In the experiment, the suspension of microparticles is prepared in a 1500 ml container. To this container is added 1 drop of microparticles at a time. The volume of one drop of microparticles has previously been determined to $4.15 \times 10^{-2}$ ml, see Table 5.1. By adding $4.15 \times 10^{-2}$ ml to the 1500 ml container the concentration of microparticles has been changed by

$$\frac{4.15 \times 10^{-2} \text{ ml}}{1500 \text{ ml}} = 2.8 \times 10^{-5}.$$\textsuperscript{1}Polysciences, Inc., Warrington, PA, USA.
Therefore the concentration $c$ (number of particles per milliliter) will be

$$c = n_d \cdot 2.8 \times 10^{-5} \cdot 1.4 \times 10^{10} \text{ particles/ml} = n_d \cdot 3.9 \times 10^5 \text{ particles/ml},$$

where $n_d$ is the number of drops added to the 1500 ml container.
Appendix B

Particles found in GRIP Ice Cores

B.1 Number Density of Particles

Steffensen [1997] found the mass of dust particles in selected segments of the GRIP ice core. In the following these masses are converted to number of particles per milliliter. Here the log-normal mode is used as the mean of the particles in the given depth interval.

Assuming a dust density of $\rho_{dust} = 2500 \text{ kg/m}^3$ the mass $m$ of 1 particle, with radius $a$ and volume $V$, is given as

$$m = \rho_{dust}V = 2500 \text{ kg/m}^3 \cdot \frac{4}{3}\pi a^3.$$

The number of particles $N_{kg}$ in 1 kg of ice is then given as

$$N_{kg} = \frac{\text{dust mass}}{m} \text{ [kg}^{-1}],$$

where dust mass [kg dust/kg ice] is the mass for the specific depth, given in Steffensen [1997], Table 1, page 26,758.

The density of ice is $\rho_{ice} = 920 \text{ kg/m}^3 = 0.92 \times 10^{-3} \text{ kg/ml}$. Therefore the number of particles per milliliter $N_{ml}$ is given as

$$N_{ml} = 0.92 \times 10^{-3} \text{kg/ml} \cdot N_{kg} \text{ [ particles/ml]}.$$

As an example consider two dust masses of $100 \times 10^{-9}$ and $7300 \times 10^{-9}$ kg dust/kg ice with mean radius $a = 1.0 \mu\text{m}$. These values are not uncommon for dust masses found in ice sheet ice from Greenland according to Steffensen [1997]. The number density of particles for the two dust masses are found in the following.

The mass of one particle with a mean radius of $a = 1.0 \times 10^{-6} \text{ m}$ is:

$$m = 2500 \text{ kg/m}^3 \cdot \frac{4}{3}\pi (1.0 \times 10^{-6} \text{ m})^3 = 1.0 \times 10^{-14} \text{ kg}$$
The number of particles per kg ice, for the two dust masses, is then found as:

\[
100 \mu g/kg : N_{kg} = \frac{100 \times 10^{-9} \text{ kg dust/kg ice}}{1.0 \times 10^{-14} \text{ kg dust}} = 1.0 \times 10^7 \text{ particles/kg ice}
\]

\[
7300 \mu g/kg : N_{kg} = \frac{7300 \times 10^{-9} \text{ kg dust/kg ice}}{1.0 \times 10^{-14} \text{ kg dust}} = 7.3 \times 10^8 \text{ particles/kg ice}
\]

The number of dust particles per milliliter ice can now be found as:

\[
100 \mu g/kg : N_{ml} = 0.92 \times 10^{-3} \text{ kg/ml} \cdot 1.0 \times 10^7 \text{ particles/kg ice} = 9.2 \times 10^3 \text{ particles/ml}
\]

\[
7300 \mu g/kg : N_{ml} = 0.92 \times 10^{-3} \text{ kg/ml} \cdot 7.3 \times 10^8 \text{ particles/kg ice} = 6.7 \times 10^5 \text{ particles/ml}
\]

or equivalently the number of particles per mm$^3$ ice

\[
100 \mu g/kg : N_{mm^3} = 920 \text{ kg/m}^3 \cdot 10^{-9} \text{ m}^3/\text{mm}^3 \cdot 1.0 \times 10^7 \text{ particles/kg ice} = 9.2 \text{ particles/mm}^3
\]

\[
7300 \mu g/kg : N_{mm^3} = 920 \text{ kg/m}^3 \cdot 10^{-9} \text{ m}^3/\text{mm}^3 \cdot 7.3 \times 10^8 \text{ particles/kg ice} = 670 \text{ particles/mm}^3
\]
Appendix C

Calculation of $\phi_{\text{eff}}$, $\psi_1$, and $\psi_2$

The effective field of view $\phi_{\text{eff}}$

The field of view $\phi$ of the camera is $31.3^\circ$ in a homogeneous medium. In the field deployed geometry the light reaching the camera has gone through several media with resulting refractions. Due to this refraction the effective field of view $\phi_{\text{eff}}$ is smaller than the field of view found in a homogenous medium. In the following the effective field of view is calculated. Refractive indices are shown in section 3.2

Starting at the lens the first refraction takes place at the air→quartz window interface inside the hull of the Cryobot, see Figure 3.4 and Figure C.1. In the following we define the angles in the same way as for the laser beam going through the media even though it is the field of view that is calculated. At the first interface $\phi/2$ is reduced to

$$\theta_{hq}^i = \arctan \left( \frac{n_{\text{air}}'}{n_q'} \sin \theta_{hq}^t \right) = 10.1^\circ,$$

where $\theta_{hq}^t$ is the transmittance angle at the hull→quartz window interface and $\theta_{hq}^i$ is the incidence angle, i.e. $\theta_{hq}^i = \phi/2 = 15.6^\circ$ for a homogeneous medium.

The next interface is the quartz window→water interface. The transmittance angle for the hull→quartz interface becomes the incidence angle for the quartz→water interface. Therefore, at this interface, the field of view changes from $\theta_{hq}^i = \theta_{qw}^i = 10.1^\circ$ to

$$\theta_{qw}^t = \arctan \left( \frac{n_q'}{n_w'} \sin \theta_{qw}^i \right) = 11.7^\circ.$$

The last interface is the water→ice interface and the effective field of view is thus given as (with $\theta_{wi}^i = \theta_{qw}^i$)

$$\phi_{\text{eff}} = 2 \cdot \arctan \left( \frac{n_w'}{n_{\text{ice}}'} \sin \theta_{wi}^i \right) = 23.8^\circ.$$
APPENDIX C. CALCULATION OF $\phi_{\text{eff}}$, $\psi_1$, AND $\psi_2$

The scattering angles $\psi_1$ and $\psi_2$

The scattering angles $\psi_1$ and $\psi_2$ are calculated in the following way, see Figure C.1. To find the scattering angle $\psi_2$ we first need to find the angles $\gamma$ and $\beta$ as

$$\psi_2 = 180^\circ - (\gamma - \beta).$$

To find $\beta$ we need to find the slope $\alpha$ of the laser beam going through the ice (this is also the laser transmittance angle into the ice $\theta_{wi}$, see Figure 3.4). To do this we find the positions of the points $p$ and $q$ relative to the point $O$. Using the angles derived in the section above and the following values: window height = 20 mm, meltwater thickness = 5 mm, min. detection distance = 10 mm, and max. detection distance = 100 mm, we find the positions of points $p$ and $q$ to be

$$p: (x,y) = (15 \text{ mm}, -13.1 \text{ mm}) \quad \text{and} \quad q: (x,y) = (105 \text{ mm}, 32.1 \text{ mm}).$$

This gives us the slope of the laser beam as

$$\alpha = \arctan \left( \frac{32.1 \text{ mm} - (-13.1 \text{ mm})}{105 \text{ mm} - 15 \text{ mm}} \right) = 26.7^\circ.$$

The two scattering angles are then found as

$$\psi_1 = 180^\circ - \phi_{\text{eff}}/2 - \alpha = 141.4^\circ,$$

$$\psi_2 = 180^\circ - (\gamma - \beta) = 180^\circ - (180^\circ - 90^\circ - \phi_{\text{eff}}/2) + (180^\circ - 90^\circ - \alpha) = 165.2^\circ.$$
Bibliography


Hecht, E. *Optics*. Addison-Wesley, Reading, Massachusetts, 2002.


BIBLIOGRAPHY


